Novel transversity properties in semi-inclusive deep inelastic scattering

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The T-odd distribution functions contributing to the transversity properties of the nucleon and their role in fueling nontrivial contributions to azimuthal asymmetries in semi-inclusive deep inelastic scattering are investigated. We use a dynamical model to evaluate these quantities in terms of HERMES kinematics.

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I. INTRODUCTION

Transversity, as combinations of helicity states for the moving nucleon, is a variable introduced originally [1] to reveal an underlying simplicity in nucleon–nucleon spin dependent scattering amplitudes. The connection with the spin structure of the nucleon in terms of the chiral odd distribution $h_1(x)$ and its first moment, the tensor charge, emerged in the analyses of hard processes [2]. These analyses revealed that to leading order in inverse powers of $Q^2$, transversity is strongly suppressed in deep inelastic scattering and the nonconservation of the tensor charge makes predicting transversity fairly difficult. Until recently, our theoretical knowledge of transversity appeared limited to suggestive bounds placed on the leading twist quark distributions in the form of the inequality of Soffer [3]. In principle, however, transversity can be probed when at least two hadrons are present (as was known earlier e.g., the Drell-Yan process [4] or in semi-inclusive deep inelastic scattering (SIDIS) [5]. These analyses inspired the first searches for the transversity properties of the nucleon [6,7]. Further, while probing transversity through exclusive single meson production has met with negative results [8], it has been suggested that the chiral odd off forward distribution can be probed when two vector mesons are observed [9]. Additionally, when quark distributions are enriched with $k_\perp$ dependence [10,11], while also allowing for time reversal odd (T-odd) behavior [12–14], novel transversity properties of quarks in hadrons can be measured via asymmetries in semi-inclusive and polarized spin processes. For example, the distribution $f_{1T}(x,k_\perp)$, representing the number density of unpolarized quarks in transversely polarized nucleons, may be entering the recent measurements of single spin asymmetries (SSAs) at HERMES [7] and at the BNL Relativistic Heavy Ion Collider (RHIC) [15] in semi-inclusive pion electroproduction. Alternatively $h_1^+(x,k_\perp)$, which describes the transfer of transversity to quarks inside unpolarized hadrons, may enter transverse momentum dependent asymmetries [13,14,16–18]. Beyond the T-odd properties, the existence of these distributions is a signal of the essential role played by the intrinsic transverse quark momentum and the corresponding angular momentum of quarks inside the target and fragmenting hadrons in these hard scattering processes. In this paper we analyze the T-odd transversity properties of quarks in hadrons which emerge in SIDIS. We apply these results to predict $\cos 2\phi$ [13] and Sivers [19] asymmetries in terms of HERMES kinematics.

II. TRANSVERSITY AND INTRINSIC TRANSVERSE MOMENTUM

From the standpoint of quark-target helicity flip amplitudes, what has emerged from analyzing transversity in inclusive, semi-inclusive, and exclusive processes is that angular momentum conservation requires that helicity changes are accompanied by transferring 1 or 2 units of orbital angular momentum, highlighting the essential role of intrinsic $k_\perp$ and orbital angular momentum in determining transversity. The interdependence of transversity on quark orbital angular momentum and $k_\perp$ is a general property. This behavior was revealed in studying the [20] vertex function associated with the tensor charge and in generalized meson production amplitudes in exchange models. In these cases, angular momentum conservation results in the transfer of orbital angular momentum $\ell = 1$ carried by the dominant $J^{PC} = 1^{+}$ mesons to compensate for the non-conservation of helicity across the vertex. The corresponding conjugate dependence on powers of the intrinsic quark momentum is determined by these tensor couplings which involve helicity flips associated with kinematic factors of 3-momentum transfer, as required by angular momentum conservation. This $k_\perp$ dependence can be understood on fairly general grounds from the kinematics of the exchange picture in exclusive pseudoscalar meson photoproduction and points to the fundamental role of rescattering as the source for nontrivial leading twist SSAs associated with transversity.

For large $s$ and relatively small momentum transfer $t$ combinations of the four helicity amplitudes involve definite parity exchanges [21]. The four independent helicity amplitudes can have the minimum kinematically allowed powers,

\[ f_1 = f_{1+0} \propto k_\perp^1, \quad f_2 = f_{1+0} \propto k_\perp^0, \]

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However, in single hadron exchange (or Regge pole exchange) parity conservation requires $f_1 = \pm f_4$ and $f_2 = \mp f_3$ for even or odd parity exchanges. These pair relations, along with a single hadron exchange model, force $f_2$ to behave like $f_3$ for small $t$. This introduces the $k_2^2$ factor into $f_2$. However for a non-zero polarized target asymmetry to arise there must be interference between single helicity flip and non-flip and/or double flip amplitudes. Thus this asymmetry must arise from rescattering corrections (or Regge cuts or eikonalization or loop corrections) to single hadron exchanges. That is, one of the amplitudes in

$$P_y = \frac{2 \text{Im}(f_1^* f_3 - f_1^* f_2)}{\sum_{j = 1 }^{4} |f_j|^2}$$

must acquire a different phase. In fact rescattering reinstates $f_2 \propto k_1^2$ by integrating over loop $k_\perp$, which effectively introduces a $\langle k_2^2 \rangle$ factor [21]. This is true for the inclusive process as well, where only one final hadron is measured; a relative phase in a helicity flip three body amplitude is required [22].

### III. T-ODD DISTRIBUTIONS IN SEMI-INCLUSIVE REACTIONS

Recently rescattering was considered as a mechanism for SSAs in pion electroproduction from transversely polarized nucleons. Using the QCD motivated quark-diquark model of the nucleon [23,24], the T-odd distribution function, $f_{1T}(x,k_\perp)$ and the corresponding analyzing power for the azimuthal asymmetry in the fragmenting hadron’s momentum and spin distributions resulted in a leading twist nonzero Sivers [19] asymmetry [14,25]. Using the approach in Ref. [25], we investigated the rescattering in terms of final state interactions (FSI) to the T-odd function $h_1^t(x)$ and corresponding azimuthal asymmetry in SIDIS [16,17]. The asymmetry involves the convolution with the T-odd fragmentation function, $h_1^t(x) \ast H^t_1(c)$ [13]. As mentioned in the introduction $h_1^t(x,k_\perp)$ is complimentary to the Sivers function and is of great interest theoretically, since it vanishes at the tree level, and experimentally, since its determination does not involve polarized nucleons [13,14,16–18].

The T-odd distributions are readily defined from the transverse momentum dependent quark distributions [27,14] where the well known identities for manipulating the limits of an ordered exponential lead to the expression

$$\Phi^{[1]}(x,k_\perp) = \frac{1}{2} \sum_n \int \frac{d\xi^+ d^2\xi_\perp}{(2\pi)^3} e^{-i(\xi^+ k^+ - \xi_\perp k_\perp)} \langle P | \bar{\psi}(\xi^- , \xi_\perp) \psi(0,0) | P \rangle \xi^+ = 0$$

and the path ordered exponential is $G^{[\infty,\xi]} \equiv \mathcal{P} \exp \left(-ig \int_\xi^{\infty} d\xi' A^+(\xi') \right)$, and $\{|n\}$ is a complete set of states. While the path ordered light-cone link operator is necessary to maintain gauge invariance and appears to respect factorization [14,25,26] when transverse momentum distributions are considered, in non-singular gauges [25,26], it also provides a mechanism to generate interactions between an eikonalized struck quark and the remaining target. These final state interactions in turn gives rise to leading twist contribution to the distribution functions that fuel the novel SSAs that have been reported in the literature [14,16–18,24–26].

The Feynman rules for eikonal lines and vertices were derived sometime ago [27,14] and applied to the T-odd Sivers function [25] and $h_1^t$ [16,17] recently. They are obtained by expanding the interactions in the path ordered gauge link operator in Eq. (3). With the tree level contribution vanishing, the leading order one loop contribution of final state interactions to the T-odd transverse quark distribution function comes from the first non-trivial term in the expansion. Modeling the remaining target in the quark-diquark model [23,25], $\Phi^{[1]}$ takes the form

$$\Phi^{[1]}(x,k_\perp) = \frac{1}{2} \sum_n \int \frac{d\xi^+ d^2\xi_\perp}{(2\pi)^3} e^{-i(\xi^+ k^+ - \xi_\perp k_\perp)} \langle P | \bar{\psi}(\xi^- , \xi_\perp) \psi(0,0) | P \rangle \xi^+ = 0$$

$$\times \left( \left| n \right| \left( -ie_1 \int_0^\infty A^+(\xi^- , 0) d\xi^- \right) \Gamma \psi(0,0) \right)$$

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where $e_1$ is the charge of the struck quark and now $\{|n\}$ represents intermediate scalar diquark spectator states. The quark-nucleon-spectator model used in previous rescattering calculations assumed a point-like nucleon-quark-diquark vertex, which leads to logarithmically divergent, $x$-dependent distributions. Yet we know there is a distribution of intrinsic transverse momenta among the constituents of the nucleon, as Drell-Yan processes show [28]. To account for this fact and to also address the log divergence [16–18,24,25] we assume the transverse momentum dependence of the quark-nucleon-spectator vertex can be approximated by a Gaussian distribution in $k_1^2$ [20]:

$$\langle n | \psi(0) | P \rangle = \frac{i}{k - m} g_z e^{-k_1^2/(k_1^2 \pi)} U(P,S)$$

where $g_z$ (defined henceforth as $g$) is the scalar diquark coupling [23,24], $k$ is the momentum of the quark in the target proton, $k_1$ and $\langle k_1^2 \rangle$ are the intrinsic and average intrinsic transverse momentum respectively, and $U(P,S)$ is the nucleon spinor. The quark propagator comes from the untruncated quark line. Going to momentum space, performing the loop integration, and finally projecting the unpolarized piece from $\Phi^{[1]}(\sigma^L \gamma_5)$ results in the leading twist, T-odd, unpolarized contribution [29].
Here, $e_2$ is the gluon-scalar diquark coupling, and $\Lambda(k_2^q) = k_1^q + (1-x)m^2 + x\lambda^2 - x(1-x)M^2$, where $M$, $m$, and $\lambda$ are the nucleon, quark, and diquark masses respectively. Also, $b = 1/(k_2^q)$, where $(k_2^q)$ is fixed below. As a check on our approach, letting $b$ go to zero which is equivalent to letting $(k_2^q) \rightarrow \infty$ and expanding the incomplete gamma function $\Gamma(0,z)$ in powers of $z = b\Lambda$, we obtain the log divergent result [16-18]. The average $k_2^q$ is a regulating scale which we fit to the expression for the integrated unpolarized structure function

$$f(x) = \frac{8^2}{(2\pi)^2} \frac{b^2}{\pi^2} (1-x) \left[ \frac{(m+xM)^2 - \Lambda(0)}{\Lambda(0)} \right]$$

$$- \left[ 2b((m+xM)^2 - \Lambda(0)) - 1 \right] e^{2b\Lambda(0)} \Gamma(0,2b\Lambda(0))$$

which multiplied by $x$ at $(k_2^q) = (0.4)^2$ GeV$^2$ is in good agreement with the valence distribution of Ref. [30].

The $T$-odd distribution is leading twist and IR finite and thus provides a phenomenological basis from which to model the fragmentation process, which along with the quark fragmentation function $\Delta(p)$ enters the hadronic tensor to leading order in $1/Q^2$ [11]

$$MW^{\mu\nu}(P,P,h,q) = \int d^4k d^4p \delta^4(k+q-p)$$

$$\times \text{Tr}(\gamma^\mu \Phi(k) \gamma^\nu \Delta(p))$$

$$+ \begin{cases} q & \rightarrow -q \\ \mu & \rightarrow \nu \end{cases}$$

$$= \int d^2P_{h\perp} \frac{|P_{h\perp}^2|}{MM_h \cos 2\phi} \cos 2\phi \quad d\sigma$$

$$= \frac{8(1-y) \sum q e_q^2 h_{1}^{(1)}(x) z^2 H_{1}^{(1)}(z)}{(1 + (1-y)^2) \sum q e_q^2 h_{1}^{(1)}(x) D_{1}(z)}$$

where the subscript $UU$ indicates unpolarized beam and target (Note: The non-vanishing $\cos 2\phi$ asymmetry originating from $T$-even distribution and fragmentation function appears only at order $1/Q^2$ [32,10,33]. Additionally, the SSA characterizing the so-called Sivers effect is
where the subscript $\text{UT}$ indicates unpolarized beam and transversely polarized target. The functions $h_{1}^{\perp}(x)$, $f_{1}^{\perp}(x)$, and $H_{1}^{\perp}(z)$ are the weighted moments of the distribution and fragmentation functions [31] where $M$ and $M_{h}$ are the mass of the target proton and produced hadron respectively. We evaluate the $\langle \cos 2\phi \rangle_{\text{UT}}$ and $\Lambda_{\text{UT}}^{\sin(\phi - \phi_{S})}$ asymmetries, under the $u$-quark dominance, obtained from the approximations

$$
\left\langle \cos 2\phi \right\rangle_{\text{UT}} \approx \frac{MM_{h}}{P_{h\perp}^{2}} \left( \frac{P_{h\perp}}{MM_{h}} \cos 2\phi \right)_{\text{UT}},
$$

$$
\Lambda_{\text{UT}}^{\sin(\phi - \phi_{S})} = 2 \frac{MM_{h}}{P_{h\perp}} \left( \sin(\phi - \phi_{S}) \right)_{\text{UT}}
$$

in the HERMES kinematic range corresponding to, 1 GeV$^{2}$ $\leq Q^{2} \leq$ 15 GeV$^{2}$, 4.5 GeV $\leq E_{x} \leq$ 13.5 GeV, 0.2 $\leq z \leq$ 0.7, 0.2 $\leq y \leq$ 0.8, and taking $P_{h\perp}^{2} = 0.25$ GeV$^{2}$ and $P_{h\perp}$ = 0.4 GeV as input. The Collins ansatz [5,31] for the analyzing power of transversely polarized quark fragmentation function $H_{1}^{\perp}(z)$, has been adopted [34]. For $D_{1}(z)$, the simple parametrization from Ref. [35] was used. In Figs. 1 and 2 the $\langle \cos 2\phi \rangle_{\text{UT}}$ and $\Lambda_{\text{UT}}^{\sin(\phi - \phi_{S})}$ of Eqs. (11) for $\pi^{+}$ production on a proton target is presented as a function of $x$ and $z$, respectively. Using $\Lambda_{QCD} = 0.2$ GeV, and $\mu = 0.8$ GeV, Fig. 1 indicates approximately a 2% $\cos 2\phi$ asymmetry and Fig. 2 a 10% $\sin(\phi - \phi_{S})$ asymmetry. In Fig. 3 we plotted the ratio of the $z$-dependence of the double $T$-odd $\cos 2\phi$ to the single spin $\sin(\phi - \phi_{S})$ asymmetry. The result is proportional to the $z$ times the analyzing power of transversely polarized quark fragmentation and reflect the equality of $h_{1}^{\perp}$ and $f_{1}^{\perp}$ functions in our approach.

V. CONCLUSION

The interdependence of intrinsic transverse quark momentum and angular momentum conservation are intimately tied with studies of transversity. This was demonstrated previously from analyses of the tensor charge in the context of the axial-vector dominance approach to exclusive meson photoproduction [20], and to SSAs in SIDIS [16,17]. In the case of unpolarized beam and target, we have predicted that at HERMES energies there is a sizable $\cos 2\phi$ asymmetry associated with the asymmetric distributions of transversely polarized quarks inside unpolarized hadrons. In order to evaluate this asymmetry we have modeled the quark intrinsic momentum with a Gaussian distribution [20]. Further we have predicted the Sivers asymmetry, and as a check on this approach, we also predict the ratio of the $\cos 2\phi$ to $\Lambda_{\text{UT}}^{\sin(\phi - \phi_{S})}$ asymmetries. This ratio is consistent with the ansatz of Collins [5].

Beyond these model calculations it is clear that final state interactions can account for SSAs. In addition, it has been shown that other mechanisms, ranging from initial state interactions to the non-trivial phases of light-cone wave functions [24,36] can account for SSAs. These various mechanisms can be understood from the context of gauge fixing as it impacts the gauge link operator in the transverse momentum quark distribution functions [25,26]. Thus using rescattering as a mechanism to generate $T$-odd distribution functions opens a new window into the theory and phenomenology of transversity in hard processes.

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[29] The details of this calculation will be presented in a longer publication; L. Gamberg, G. R. Goldstein, and K.A. Oganessyan in preparation.