A-dependence of hadronization in nuclei

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The study of hadronization, the process that leads from partons produced in some elementary interaction to the hadrons observed experimentally, is of importance, both in its own right as a study of a nonperturbative QCD process and in the interpretation of data from experiments that use outgoing hadrons as a tag. The end products of the hadronization process in free space are known from hadrons as a tag. The end products of the hadronization process in the interpretation of data from experiments that use outgoing its own right as a study of a nonperturbative QCD process and hadrons observed experimentally, is of importance, both in partons produced in some elementary interaction to the hadrons.

The A-dependence of models for the attenuation of hadron production in semi-inclusive deep-inelastic scattering on a nucleus is investigated for realistic matter distributions. It is shown that the dependence for a pure partonic (absorption) mechanism is more complicated than a simple $A^{2/3} (A^{1/3})$ behavior, commonly found when using rectangular or Gaussian distributions, but that the A-dependence may still be indicative for the dominant mechanism of hadronization.

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Because hadronization, as a nonperturbative QCD process, cannot be calculated from first principles, various models have been developed (see, e.g., Refs. [4–9]) to describe hadron production and attenuation in a nucleus. Some models focus on the partonic part, whereas others include or emphasize the hadronic part. In all cases a sizable dependence on the mass number $A$ is predicted. However, often the calculations use simple forms for the matter distribution of the nucleus.

In this Brief Report we investigate the A-dependence using realistic matter distributions. We do this for two schematic models, covering the extremes sketched above. The first one (called model I) assumes a purely partonic mechanism, hadronization occurring outside of the nucleus (point B in Fig. 1 effectively at infinity) and thus absorption of the produced hadrons playing no role. The nuclear attenuation is then caused by the energy loss of the produced quark due to multiple scattering and gluon bremsstrahlung, which gives rise to a change of the hadron fragmentation function and thus to nuclear attenuation. Because of the Landau-Pomeranchuk-Migdal interference effect [10] this energy loss depends quadratically on the length of matter traversed by the hit quark (for the present discussion we neglect contributions from other quarks; see Refs. [6,7] for more details). In terms of the picture of Fig. 1 it depends on the square of the density-averaged distance the parton travels within the nucleus from the point where it is created (point A in Fig. 1).

On the other hand, our second schematic model (called model II) assumes that a possible attenuation is completely due to absorption of the produced hadron, so nothing happens between the time the parton is produced and the hadron is formed (points A and B in Fig. 1). In a Glauber approach this attenuation depends on the cross section for absorption of the hadron3 and the density-averaged distance the hadron travels through the nucleus after it has been formed (see Fig. 1). Taking the absorption cross section to be constant (and small enough that a linear approximation is sufficient), the latter determines the nuclear attenuation.

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1For a more detailed discussion of the concept of formation time, etc., see Ref. [3].
2For the present discussion it is not needed to discriminate between hadrons and prehadrons, so in the remainder we discuss only hadrons (but see, e.g., Refs. [4,5]).
3In our simple model we do not take into account coupled-channels processes. Possible effects of these are discussed in Ref. [5].
This means we have to calculate the following two integrals:

\[
\langle t_I^2 \rangle = \frac{2\pi}{A} \int_{0}^{\infty} \int_{-\infty}^{\infty} b d b \int_{-\infty}^{\infty} d z \rho_A(b, z) \times \left[ \int_{-\infty}^{\infty} d z' \rho_A(b, z') \right]^2,
\]

(model I) \hspace{1cm} (1)

\[
\langle t_{II} \rangle = \frac{2\pi}{A} \int_{0}^{\infty} \int_{-\infty}^{\infty} b d b \int_{-\infty}^{\infty} d z \rho_A(b, z) \times \int_{0}^{\infty} d z' L_f^{-1} e^{-(z' - z)/L_f} \times \int_{0}^{\infty} d z'' \rho_A(b, z'')\hspace{1cm} (\text{model II}) \hspace{1cm} (2)
\]

Here the exponential models the distribution of the formation distances \(L_f\), and the matter densities \(\rho_A\) are normalized to \(A\). (By entering these quantities with corresponding dynamical factors into the appropriate formula’s of the models, hadron production cross sections and from these values for the attenuation \(R_A\) can be calculated. However, here we are interested in the \(A\)-dependence (and, moreover, the used models are extreme and schematic). Under the assumption that the cross section is linear in \(\langle t_I^2 \rangle\) and \(\langle t_{II} \rangle\), which is a good first-order approximation, the \(A\)-dependence of these quantities carries over into the one of the cross sections.)

It can easily be shown that for a nucleus with a mass density distribution described by one scale parameter, as in the case of a rectangular (as in the liquid-drop model) or a Gaussian distribution, the value of \(\langle t_I^2 \rangle\) is proportional to the equivalent radius or rms radius squared, which in those cases leads to an \(A^{2/3}\)-dependence. In case of model II one finds for \(\langle t_{II} \rangle\) for a rectangular distribution an \(A^{1/3}\)-dependence when \(L_f = 0\), and a larger exponent\(^4\) (e.g., about 0.55 for \(L_f = 4\ fm\)) at larger \(L_f\), and similarly for a Gaussian distribution.

However, neither a rectangular nor a Gaussian distribution is a good representation of the mass distribution of a real nucleus. Therefore we have evaluated \(\langle t_I^2 \rangle\) and \(\langle t_{II} \rangle\) for a more realistic distribution, described by a two-parameter Fermi

\[\rho_A(r) = \rho_0 / [1 + e^{-(r-r_0)/a}] \hspace{1cm} (3)\]

with parameters \(\rho_0 = 0.170\) nucleons/fm\(^3\), \(a = 0.5\) fm, and the value of \(c\) so as to give a nucleus with \(A\) nucleons. (This form gives a reasonably good global description of the mass distribution down to low values of \(A\)). The results are given in Table I and are shown in Figs. 2 and 3.

It can be seen from Fig. 2 that for a 2pF matter distribution the \(A\)-dependence of \(\langle t_I^2 \rangle\) is rather steep. If one tries to describe it with the power law \(A^\alpha\), it would require a value of \(\alpha\) or well in excess of 2/3. Figure 3 shows for the case of model II that the

\[\text{FIG. 2. Values of } \langle t_I^2 \rangle \text{ as function of } A \text{ for two-parameter Fermi (2pF) matter distributions (solid curve) and for actual nuclei (circles). The dashed line is a power-law (} A^\alpha \text{) fit to the latter, excluding the point for the deuteron. The dotted line in addition excludes the point for } ^4\text{He in the fit.}\]

\[\text{FIG. 1. Illustration of hadronization in a nucleus and used coordinates. The parton is produced at point A, whereas the hadron is formed at B. The distance } L_f \text{ has a distribution with as average the formation length } L_f.\]

\[\text{TABLE I. Values of } \langle t_I^2 \rangle \text{ and } \langle t_{II} \rangle \text{ for nuclei of mass number } A, \text{ calculated with a 2pF distribution [see Eq. (3)] with half radius } c \text{ (second column) and } \rho_0 = 0.170\ \text{fm}^{-3} \text{ and } a = 0.5\ \text{fm} \text{ fixed. The values of } \langle t_{II} \rangle \text{ were calculated for five different values of } L_f \text{ (columns 4–8).}\]

\[
\begin{array}{cccccccc}
A & c & \langle t_I^2 \rangle & (\text{fm}^{-4}) & \langle t_{II} \rangle & (\text{fm}^{-2}) \\
\hline
4 & 1.321 & 0.023 & 0.108 & 0.070 & 0.048 & 0.037 & 0.037 & 0.030 \\
8 & 1.875 & 0.052 & 0.166 & 0.113 & 0.081 & 0.062 & 0.051 & 0.049 \\
16 & 2.531 & 0.106 & 0.243 & 0.175 & 0.129 & 0.101 & 0.083 & 0.077 \\
32 & 3.324 & 0.200 & 0.334 & 0.264 & 0.260 & 0.199 & 0.159 & 0.132 \\
64 & 4.295 & 0.359 & 0.468 & 0.373 & 0.295 & 0.242 & 0.204 & 0.184 \\
128 & 5.452 & 0.609 & 0.619 & 0.513 & 0.421 & 0.353 & 0.303 & 0.271 \\
256 & 6.967 & 1.037 & 0.818 & 0.703 & 0.597 & 0.513 & 0.448 & 0.400 \\
\end{array}
\]

\[\text{4This effect of taking into account a distribution of formation distances has been noted already in Ref. [8].}\]
slope of the curve, in essence the value of $\alpha$, increases when $L_f$ increases. Values range from about $\alpha = 0.40$ for $L_f = 0$ fm to $\alpha = 0.60$ for $L_f = 4$ fm.

Given these findings, and since it is known that the parameters for actual nuclei are slightly irregular because of, e.g., shell closures, it is interesting to see the behavior of $\langle t_{II} \rangle$ and $\langle t_{I} \rangle$ for real nuclei, since those are used in experiments. For that purpose we have used parametrizations [11,12] of measured charge distributions (because the neutron distribution is very similar the error introduced by using the charge distribution instead of the matter distribution is small). For actual nuclei (symbols) the error introduced by using the $\langle t_{II} \rangle$ fits, as indicated.

Given these results it would in principle be possible to discriminate on account of the $A$-dependence between the two extreme mechanisms that we have used here. However, in practice the process of hadronization in a nucleus most probably will be a combination of these mechanisms, with possibly even different dependences on path lengths in the nucleus than employed here. But whatever the mechanism, the $A$-dependence will be an important ingredient. Our results show that in model calculations of the $A$-dependence for real nuclei (symbols), the results for $2pF$ model for actual nuclei (symbols) and $2pF$ model for real nuclei, including $^4$He (but leaving out $^3$H), one finds an exponent of $0.74$ for $\langle t_{II}^2 \rangle$ (see Fig. 2) and values between $0.40$ and $0.60$ for $\langle t_{II} \rangle$ depending on the value of $L_f$ (see Fig. 4).

When $^4$He is not included in the fit, the values of $\alpha$ in the case of $\langle t_{II} \rangle$ change by less than $0.02$, whereas in case of $\langle t_{II} \rangle$ the value of $\alpha$ increases from $0.74$ to $0.79$. Thus, in trying to extract an $A$-dependence from experimental data it matters if one uses $^4$He as lowest nucleus or, e.g., $^{12}$C.

TABLE II. Calculated values of $\langle t_{II}^2 \rangle$ and $\langle t_{II} \rangle$ for various nuclei.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$r_{rms}$</th>
<th>$\langle t_{II}^2 \rangle$</th>
<th>$\langle t_{II} \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[fm]</td>
<td>[fm$^{-2}$]</td>
<td>[fm$^{-2}$]</td>
</tr>
<tr>
<td>$^2$H</td>
<td>2.13</td>
<td>0.111</td>
<td>0.068</td>
</tr>
<tr>
<td>$^4$He</td>
<td>1.72</td>
<td>0.050</td>
<td>0.159</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>2.45</td>
<td>0.086</td>
<td>0.218</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>2.73</td>
<td>0.101</td>
<td>0.238</td>
</tr>
<tr>
<td>$^{28}$Si</td>
<td>3.08</td>
<td>0.173</td>
<td>0.316</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>3.47</td>
<td>0.216</td>
<td>0.356</td>
</tr>
<tr>
<td>$^{48}$Ca</td>
<td>3.77</td>
<td>0.309</td>
<td>0.428</td>
</tr>
<tr>
<td>$^{84}$Kr</td>
<td>4.25</td>
<td>0.395</td>
<td>0.493</td>
</tr>
<tr>
<td>$^{118}$Sn</td>
<td>4.67</td>
<td>0.523</td>
<td>0.571</td>
</tr>
<tr>
<td>$^{132}$Xe</td>
<td>4.83</td>
<td>0.570</td>
<td>0.598</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>5.51</td>
<td>0.811</td>
<td>0.719</td>
</tr>
</tbody>
</table>

FIG. 4. Values of $\langle t_{II} \rangle$ as function of $A$ for $L_f = 0, 2, 4$ fm for real nuclei (symbols). The lines are results of power-law ($A^\alpha$) fits, excluding the point for the deuteron.
such as a rectangular distribution employed in the liquid-drop model, are not adequate. Instead, the use of experimentally established density functions for the nuclei actually used in the experiment is essential, where it even matters whether $^4\text{He}$ with its relatively large density is included.

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