TMC - Vertex Reconstruction in the Presence of the HERMES Transverse Target Magnet

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1 Introduction

In the spring of 2002, a magnet providing a transverse holding field was installed in the target region of the Hermes experiment. The purpose is to provide target protons polarized transverse to the beam direction, to allow the measurement of the transverse spin structure of the nucleon. In all previous Hermes data-taking, the target nucleons and nuclei had been longitudinally polarized, i.e. in a direction parallel to the beam direction.

The magnetic force $\vec{F}$ exerted by an external magnetic field $\vec{B}$ on a beam particle of charge $q$ moving with velocity $\vec{v}$ is given by the Lorentz formula

$$\vec{F} = q\vec{v} \times \vec{B}$$

In the case of the longitudinally polarized target, the external magnetic field $\vec{B}$ is parallel to the beam direction $\vec{v}$, so there is no effect of the magnetic field on the electron/positron beam direction. Moreover, since the angular acceptance of the Hermes spectrometer is only about $\pm 140$ mrad vertically and $\pm 180$ mrad horizontally, all accepted scattered particles are traveling nearly parallel to $\vec{B}$ and thus suffer very little magnetic deflection.

The situation changes dramatically for charged particles when the longitudinal magnetic field is replaced by a transverse magnetic field $\vec{B}$. Now both the beam and the particles in the angular acceptance of the spectrometer are moving nearly perpendicular to $\vec{B}$ and suffer large, momentum-dependent magnetic deflections. In order to determine the important kinematic variables $Q^2$ and $x_{Bj}$, the azimuthal angles and the invariant mass of the hadronic final state in a deep-inelastic electron scattering event, it is necessary to accurately measure both the momentum of the outgoing lepton and its angle at the scattering vertex. The lepton momentum is accurately determined by the Hermes spectrometer, essentially by measuring the angular deflection of the track in the magnetic field. The scattering angle at the vertex must be determined from the track positions in the Front Chamber (FC) and the Drift Vertex Chamber (DVC), and a correction must be applied to account for how much the trajectory has been deflected by the transverse target magnet between the vertex point and the FC and DVC.
This report describes two algorithms for calculating the vertex angle and position for each track at the interaction point, before the track has been deflected by the transverse field $\vec{B}$. They were written at TRIUMF, Vancouver, and at the Soltan Institute for Nuclear Studies, Warsaw. Both algorithms are designed as Fortran-77 subroutines, which are called from a new Hermes software tool - TMC (Target Magnet Correction). They use as inputs the magnetic-field maps of the various transverse-target setups, the momenta of the tracks, and their absolute positions and slopes with respect to the beam in the front region, usually provided by the HERMES track reconstruction HRC. Both algorithms rely on a well-aligned detector, i.e., the beam should be "seen" at $(0, 0, z)$ in the "eyes" of the detector. The algorithms and both the implementation of the reconstruction methods as well as the usage of TMC will be described. We will briefly introduce the used field maps which are needed for preparing the reconstruction algorithms but also to test the the methods by means of the Hermes Monte Carlo simulation HMC.

2 Transverse Target Field Map

In order to prepare the correction algorithms, it is necessary to specify the magnetic field $\vec{B}$ at each point where the particle trajectory goes. Unfortunately, until Summer 2003, there was no comprehensive empirical field map for the transverse target magnet. There was an extensive set of measurements taken only along the $z$-axis, i.e., the beam axis. A few points had been taken along the $x$-axis, i.e., the horizontal direction, but extending only over a few centimeters, because their purpose was merely to verify that the magnetic field was sufficiently uniform in the region of the target gas.

Since the field strength is also needed at other points we used the theoretical field map calculated using the magnetostatic program MAFIA. Figure 1a plots the MAFIA calculated field (solid line) versus the measured data points along the $z$-axis. It is evident that the MAFIA field is too strong. If we scale the MAFIA field by a factor of 0.92397, the field strength in the interior region is correct, but the effective length of the magnet is still too small (Fig. 1b). If we now stretch the MAFIA model in the $z$ direction by a factor of 1.0435, we have a good agreement between the data and the MAFIA model (Fig. 1c).

Figure 1d shows the MAFIA field strength along the $x$-axis. There are huge computational artifacts at approximately $x=-24$ and $x=24$ cm, possibly due to the edge of the steel. Fortunately no useful particles pass through this region. Since there were no measurements along the $x$-axis, except for a very small region near the $z$-axis, it was impossible to ascertain the shape of the field in this direction. We simply assumed for the time being that the MAFIA calculation accurately reproduces the shape of the field in the $x$-direction, although the deficiencies in reproducing the effective length of the magnet in the $z$-direction might make one suspect otherwise.

This "scaled and stretched" MAFIA field was our best guess as to what the real transverse target magnet’s field look like. After the 2002/03 data-taking period the magnet’s field map has been measured again, which was also necessary as modifications, which were necessary for the functioning of the Lambda Wheels, had been done on the design. These changes are discussed in more detail in Appendix C.

3 HMC Data - a Playground for the Reconstruction Algorithms

In order to test the algorithms discussed here it was necessary to have some measure of comparing the reconstructed values with the original values of the vertex kinematics. The perfect tool for this...
was the Hermes Monte Carlo (HMC). Above-mentioned field map (see also Fig. 2) were implemented to simulate the effect on the tracks caused by the transverse target holding field. A set of data generated by HMC is found in /user/gschnell/TRANS_TARGET/MC_DATA/testfile3. This file has tracks of various types of particles, which can be fed to the two reconstruction algorithms for testing. The entries in each record are as follows:

1. Run number
2. Event number
3. MC weight
4. Lund particle type
5. generated momentum [GeV/c] - negative means negatively charged particle
6. generated polar angle $\theta$ [radians]
7. generated azimuthal angle $\phi$ [radians]
8. generated vertex position $z$ [cm]
9. reconstructed momentum [GeV/c]
10. $X$ position at FC2 [cm], located at $z=165$ cm downstream of target center
Figure 2: The stretched and scaled MAFIA map used in the HMC simulation and the correction method. Shown are isocontours in the x-z-plane (y = 0; levels given in the figure).

11. X position at DVC [cm], located at z=110 cm downstream of target center
12. Y position at FC2 [cm]
13. Y position at DVC [cm]

Another production with the latest field map (2003++) has been used and can be found at /mcdata03/DATA/DISNG_TTM. This production consists of four setups of beam misalignment conditions, described in more detail in the corresponding gnclogon kumacs and summarized in Table 1. In particular, 05_B has been used (despite having a slight beam y-slope) in order to study resolutions.

Table 1: Beam parameters of the various HMC productions in /mcdata03/DATA/DISNG_TTM/.

<table>
<thead>
<tr>
<th>Beam parameter</th>
<th>04_A</th>
<th>04_B</th>
<th>05_A</th>
<th>05_B</th>
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<td>-0.15mrad</td>
<td>0.5mrad</td>
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<tr>
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<tr>
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<td>0</td>
<td>0.12cm</td>
<td>0</td>
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</table>

4 A Transfer-Matrix based Correction Algorithm

This section describes an algorithm written by Stan Yen at TRIUMF for calculating the vertex angle and position for each track at the interaction point, before the track has been deflected by
the transverse magnetic field \( \vec{B} \). Later in this report, this is referred to as "Stan’s method" or correction method 2.

4.1 Algorithm

We will discuss the inner workings of the algorithm in some detail. The casual user who is unconcerned about the workings of the algorithm may skip this section.

4.1.1 Coordinate systems and transfer functions

![Transfer function coordinate systems](image)

The concept of a transfer function is commonly used in discussion of ion-optical systems. We use the same notation as employed by the widely-used ion-optical programs TRANSPORT and MIT-RAYTRACE. Refer to Fig. 3. Suppose that a particle has initial coordinates

\[
\vec{x}_i = (x_i, \theta_i, y_i, \phi_i, \delta_i),
\]

Figure 3: Transfer function coordinate systems.

where

- \( x_i \) is the initial horizontal position of the particle (cm),
- \( \theta_i \) is the initial horizontal angle of the particle (mrad); this corresponds to the angle \( \theta_x \) in the normal Hermes coordinate system,
- \( y_i \) is the initial vertical position of the particle (cm),
- \( \phi_i \) is the initial vertical angle of the particle (mrad); this corresponds to the angle \( \theta_y \) in the normal Hermes coordinate system,
• \( \delta_i \) is the momentum deviation (in percent) of the particle relative to the reference particle, i.e., \( \delta_i = 100(p - p_0)/p_0 \).

In this coordinate system, the reference particle, labeled “0”, has initial coordinates given by \( \vec{x}_i = (0, 0, 0, 0, 0) \). At some point further downstream, after some arbitrary optical elements (electric and magnetic dipoles, quadrupoles, etc.) we define a final coordinate system with axes \( x_f, y_f, z_f \), such that the reference particle has final coordinates \( \vec{x}_f = (0, 0, 0, 0, 0) \). Now consider another particle, labeled “1”, whose initial coordinates are close to those of the reference particle. That means \( x_1, \theta_i, y_i, \phi_i \) and \( \delta_i \) for particle 1 are all small. It is reasonable to expect that its final coordinates will be close to those of the reference particle, and we can thus do a Taylor-series expansion

\[
x_{f1} = x_{f0} + (x/x)x_{i1} + (x/\theta)\theta_{i1} + (x/\delta)\delta_{i1} + \cdots
\]

The Taylor series expansion coefficients \( (x/x), (x/\theta), (x/\delta) \), etc. are called the “transfer coefficients”, “transfer matrix elements” or “aberration coefficients”. Similar expansions may be obtained for \( \theta_{f1}, y_{f1} \), etc.

### 4.1.2 MIT-RAYTRACE

In order to evaluate the transfer coefficients, it is necessary to track some particles through the optical system in question. One widely-used computer program which does this is MIT-RAYTRACE [1]. This program allows one to specify a configuration of magnetic and electric elements, either by choosing from a set of pre-programmed elements, or by giving the program an empirical field map. The program then tracks a set of 46 particles (extended to 47 by S. Yen) at each of 5 momenta through the optical system, by numerically integrating the equations of motion via a fourth-order Runge-Kutta algorithm. The initial positions and angles of these particles are judiciously chosen so that the transfer coefficients, which are just partial derivatives of the final coordinates with respect to the initial coordinates, may be evaluated numerically by taking differences of final coordinates of different particles. The numerical integrations are computationally intensive, but need to be done only once. Once the transfer coefficients are evaluated, they provide a way of quickly relating the final coordinates \( \vec{x}_f \) to the initial coordinates \( \vec{x}_i = (x_i, \theta_i, y_i, \phi_i, \delta_i) \), i.e., we have the set of 69 coefficients defining the transfer function \( T \) such that

\[
\vec{x}_f = T[\vec{x}_i].
\]

Once the transfer coefficients have been computed, evaluating the transfer function \( T \) is MUCH faster than the Runge-Kutta numerical integration. This speed comes with a loss of information. The transfer function \( T \) contains less information than the Runge-Kutta numerical integration, because the latter computes the position of the particle at all points along its trajectory, whereas the former gives the particle’s position ONLY at the endpoint.

### 4.1.3 Transfer coefficient calculations

We embed the “scaled and stretched” MAFIA field in MIT-RAYTRACE by defining a new device type ‘TMAG’ (analogous to the existing standard devices like ‘DIPOLE’ and ‘MULTIPOLE’), such
that at each point of the particle’s trajectory, 'TMAG' retrieves the B-field components due to the MAFIA field and uses these to integrate the equations of motion for the particle.

The coordinate system used is shown in Fig. 4. We define a “RAYTRACE Reference Plane” at \( z=100 \) cm downstream from the target center. We calculate the transfer coefficients for particles starting from, say, \( z=0 \), to the reference plane at \( z=100 \) cm. Since the transfer coefficients are momentum-dependent, we evaluate the set of 69 transfer coefficients at 89 different momenta ranging from 100 MeV/c to 99 GeV/c.

Furthermore, since the target is not a thin foil confined to the plane \( z=0 \), but is in fact an extended gas jet extended from \( z=-20 \) cm to \( z=+20 \) cm, it is necessary to calculate the transfer coefficients for rays starting from \( z=-20 \) cm, \( z=-15 \) cm, \( z=-10 \) cm, \( z=-5 \) cm, \( z=0 \), \( z=5 \) cm, \( z=10 \) cm, \( z=15 \) cm, and \( z=20 \) cm. Because the trajectories of particles emanating from, for example, \( z=-20 \) cm, could go into the non-uniform region of the transverse magnet (and the uniform field region is very small in the \( x \) direction), the transfer coefficients for particles originating from \( z=-20 \) cm are not related in any simple way to the transfer coefficients for particles originating from \( z=0 \) cm. We therefore calculate 69 coefficients \( \times 89 \) momenta \( \times 9 \) values of \( z \). These sets of transfer coefficients are stored in the subroutines of file mitray_tmag3.F. The MIT-RAYTRACE program has been automated so that it could do all these calculations with minimal operator assistance.

4.1.4 Vertex reconstruction method

We denote by \( T \) the transfer function. It is defined by the set of 69 transfer coefficients. It is also a function of tabulated momentum \( P \) and the \( Z \)-plane from which the particles are assumed to originate, so we write \( T[P,Z] \). Here \( P \) is one of the 89 discrete momentum values, and \( Z \) is one of the set \(-20, -15, -10, -5, 0, 5, 10, 15, 20 \) cm at which the transfer coefficients have been calculated using
MIT-RAYTRACE. In the following discussion the upper-case $P$ and $Z$ always denote the particular discrete values of momentum and longitudinal position used in constructing the tables of transfer coefficients, whereas lower-case $p$ and $z$ denote the more general case of a particle’s momentum and longitudinal position. Operationally, given an initial vector of coordinates at $z=Z$,

$$\vec{x}_i = (x_i, \theta_i, y_i, \phi_i, \delta_i),$$  \hspace{1cm} (5)$$

the function $T$ computes a final vector of coordinates at $z=100$ cm:

$$\vec{x}_f = (x_f, \theta_f, y_f, \phi_f, \delta_f).$$  \hspace{1cm} (6)$$

So we can write

$$\vec{x}_f = T[P, Z; \vec{x}_i].$$  \hspace{1cm} (7)$$

For each track, the following is done to evaluate the vertex position and angles:

1. The track’s $(x, y)$ position in the DVC and FC, and its momentum $p$ are obtained from HRC.
2. The adjacent two tabulated momenta $P_1$ and $P_2$, such that $P_1 \leq p \leq P_2$, are obtained.
3. We don’t know beforehand the $z$-value of the vertex, so we make an initial guess of $Z = 0$. The transfer functions $T[P_1, Z = 0]$ and $T[P_2, Z = 0]$ are obtained.
4. The transfer functions $T$ are non-linear functions of the initial (at $z=0$) particle coordinate vector $\vec{x}_i$, which allow us to easily compute the particle coordinate vector $\vec{x}_f$ at the Reference Plane ($z=100$ cm). But this is the reverse of what we need to solve the problem. In the experiment, the positions $(x,y)$ in the DVC and the FC give us $\vec{x}_f$, and we want to calculate the initial vertex coordinates $\vec{x}_i$. If $T$ were a linear function $L$, then it would be easy. In that case, if we write $\vec{x}_i$ and $\vec{x}_f$ as column vectors, $L$ is just a $5 \times 5$ matrix, and we can write

$$\vec{x}_f = L \vec{x}_i,$$  \hspace{1cm} (8)$$

from which we obtain

$$\vec{x}_i = L^{-1} \vec{x}_f.$$  \hspace{1cm} (9)$$

Finding the inverse matrix $L^{-1}$ is easy. In fact, it can be shown that to first order, the coordinates $x, \theta$ and $\delta$ decouple from $y$ and $\phi$, so that $L$ is a block diagonal matrix. An additional simplification comes about if we assume that, to a good approximation, the particle momentum $p$ is the same as one of the tabulated values $P_1$ or $P_2$, so that the momentum deviation $\delta$ is identically zero. This is justified because the main effect of a momentum difference is in changing the final position and angles in the $x$-$z$ plane. The position $x_{f0}$ and the angle $\theta_{f0}$ for particles of momentum $p$ are interpolated from the values calculated by MIT-RAYTRACE for the neighboring tabulated momenta $P_1$ and $P_2$. $L$ then reduces to two $2 \times 2$ matrices along the diagonal, and computing $L^{-1}$ is simply done by inverting two $2 \times 2$ matrices, which is very easy indeed.

So, instead of the full non-linear transfer function $T$, we drop all the non-linear terms, and retain only the linear terms to get linear transfer function $L$. This allows us to get a first approximation to the vertex coordinates by

$$\vec{x}_{i1} = L^{-1} \vec{x}_f.$$  \hspace{1cm} (10)$$

We do this for the linear transfer function $L$ evaluated at the two adjacent momenta $P_1$ and $P_2$, and interpolate with momentum to get the best estimate for $\vec{x}_{i1}$. This takes care of the momentum-dependence of the transfer function.
5. We now take the approximate initial coordinate vector \( \vec{x}_{i1} \) and trace the particle forwards using the FULL transfer function \( T \), to calculate what the final coordinate vector would be.

\[
\vec{x}_{f1} = T[P, Z; \vec{x}_{i1}]
\] (11)

The difference between the calculated final coordinate vector \( \vec{x}_{f1} \) and the true final coordinates at the Reference Plane is the error vector \( \Delta x_f \). The previous step is actually done at the two adjacent momenta \( P_1 \) and \( P_2 \) and the results are interpolated in momentum to obtain the error vector \( \Delta x_f \).

6. In the linear approximation, the error in the final coordinates \( \vec{x}_f \) is related to the error in the initial coordinates \( \vec{x}_i \) by

\[
\Delta \vec{x}_f = L \Delta \vec{x}_i,
\] (12)

from which we obtain the correction to the initial coordinates

\[
\Delta \vec{x}_i = L^{-1} \Delta \vec{x}_f
\] (13)

and hence a better approximation to the initial coordinates

\[
\vec{x}_{i2} = \vec{x}_{i1} + \Delta \vec{x}_i.
\] (14)

This procedure of obtaining an approximate \( x_i \), using the full transfer function \( T \) to obtain the corresponding approximate \( x_f \), computing an error vector \( \Delta x_f \), and using the inverse linear transfer function \( L^{-1} \) to correct \( \vec{x}_i \), is iterated until convergence is achieved.

![Figure 5: Changing local gradients with successive approximations 0, 1, 2. The surface is defined by the coefficients of the full transfer function T.](image)

7. There is an additional refinement which speeds up convergence. Each of the first order transfer coefficients which define the matrix \( L \) can be thought of as the local gradient of a surface in hyperspace. For example, the coefficient \( (x/x) \) is the local gradient \( \partial x_f / \partial x_i \) and \( (x/\theta) \) is the local gradient \( \partial x_f / \partial \theta_i \). This is illustrated in Fig. 5. The tabulated values of the transfer
coefficients \((x/x)\) and \((x/\theta)\) are evaluated at the point \(x_i = 0, \theta_i = 0\), i.e., the local gradients at point 0’ on the surface in Fig. 5. However, as successive iterations carry us to points 1, 2, ..., the local gradients at the corresponding points 1’, 2’, ... on the surface. At each step, we always evaluate the LOCAL values of the gradients. We can do this because we have the full Taylor series expansion which defines the shape of the surface at all points. As an example, from Eq. 3, we obtain that the local value of the coefficient \((x/x)\), at point \((x_i, \theta_i)\), is

\[
\frac{(x/x)_{\text{local}}}{\partial x f/\partial x_i} = (x/x) + (x/x\theta)\theta_i + \cdots
\]

(15)

8. Suppose the above algorithm has converged to a set of target coordinates \((x_i, \theta_i, y_i, \phi_i)\). These are the values measured at some particular \(x\)-\(y\) plane, initially the \(z=0\) plane in Fig. 4, which is the \(z\)-plane at which we got the transfer coefficients. But this is not necessarily the vertex point. In Fig. 4, for example, the curved particle track is seen to originate not at \(z=0\), but at \(z=-12\) cm. Once we know the position and angle in the \(z=0\) plane, however, we can track the particle backwards by assuming that the track is a circular arc. The projections of the trajectory on the \(x\) and \(y\) planes are shown in Fig. 6.

![Figure 6: Vertex reconstruction by circular arcs.](image)

In general, the \(z\)-value at which \(y = 0\) (bottom frame of Fig. 6) may not be the same as the \(z\)-value where the circular arc makes the closest approach (top frame of Fig. 6).

It can be shown that, in the \(z\)-\(x\) plane, the equation of the circular arc trajectory is

\[
x = x_c - \hat{q}\sqrt{R^2 - (z-z_c)^2},
\]

(16)

where \((x_c, z_c)\) is the location of the center of the circular arc, \(R\) is the radius of the arc, and \(\hat{q}\) is the sign of the charge of the particle. The radius \(R\) is given by

\[
R \quad [m] = 33.356408 \frac{p \quad [GeV/c]}{q \quad B \quad [kGauss]},
\]

(17)

with \(q\) being the particle’s charge in units of the elementary charge. The coordinates of the center \((z_c, x_c)\) are given by

\[
\begin{align*}
z_c &= z_i - \hat{q}R \sin \theta_i \\
x_c &= x_i + \hat{q}R \cos \theta_i.
\end{align*}
\]

(18) (19)

\(^2\text{Here, } z_i \text{ is the } z \text{ value for the set of target coordinates } (x_i, \theta_i, y_i, \phi_i), \text{ usually equal to zero.}\)
In the $y$-$z$ plane, the equation of the trajectory is
\[ y = y_i + \tan \phi (z - z_i). \] (20)

The distance $D$ of each point on the trajectory from the beam axis is given by
\[ \Delta = D^2 = x^2 + y^2 = \left( x_c - \hat{q} \sqrt{R^2 - (z - z_c)^2} \right)^2 + (y_i + \tan \phi (z - z_i))^2. \] (21)

The minimum distance of approach to the beam axis is given by setting $d\Delta/dz = 0$, or
\[ 0 = \hat{q} (z - z_c) \left( x_c - \hat{q} \sqrt{R^2 - (z - z_c)^2} \right) \sqrt{R^2 - (z - z_c)^2} + \tan \phi (y_i + \tan \phi (z - z_i)). \] (22)

This equation is solved by bisection, by searching for a value of $z$ such that the function is zero. We define a bisection search interval by first taking the current $z$-value (initially 0) ±5 cm; if the function does not change sign in this interval, then we try ±10 cm, and so on, until ±25 cm is reached, which is already larger than the length of the target. Only rarely does the program fail to find a suitable bisection interval.

9. Once $z_{\text{min}}$, the $z$-value corresponding to the point of minimum approach to the beam axis is found, we go back to step 3, and repeat with the transfer coefficients evaluated not at $Z = 0$, but at the $Z$ value corresponding most closely to the point of closest approach found in the previous step. In other words, instead of using transfer function $T[P_1,Z=0]$, we use $T[P_1,Z = Z_{\text{min}}]$, where $Z_{\text{min}}$ is the value from the discrete set -20, -15, · · · , 20 cm closest to $z_{\text{min}}$. This iteration of values of $Z$ is repeated until convergence is achieved. In practice, only two iterations are usually sufficient.

4.2 User Interface

In order to use this reconstruction algorithm one needs to make a single subroutine call to reconstruct the vertex angle and position of each track:

\[
\text{call mitray\_tmag\_call(xdvc, ydvc, xfc, yfc, pmom, method, iprint, theta, phi, z, x, y, R, ierr)}
\]

The INPUT parameters are shown in Table 2. The values of the input parameters are not changed by the subroutine. The OUTPUT parameters are shown in Table 3.

5 Location of the files

The original Fortran-77 source files for this reconstruction method can be found on the Hermes PC farm in directory /afs/desy.de/user/s/stan/distr3/ (also /user/stan/distr3/ and, at TRIUMF, on hermespc4.triumf.ca:/home/stan/distr3/). The relevant files are:

- mitray\_tmag1.F (*\text{.o})
  This contains the user interface subroutine mitray\_tmag\_call that is called from your application program.
Table 2: Input parameters to subroutine mitray_tmag_call

<table>
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<tr>
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<td>cm</td>
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<tr>
<td>iprint</td>
<td>integer</td>
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<td></td>
</tr>
</tbody>
</table>

- **mitray_tmag2.F (*.o)**
  This contains most of the computational subroutines used to do the traceback to the vertex point.

- **mitray_tmag3.F (*.o)**
  This contains the transfer matrix elements from the various target z-planes ($z=-20\text{cm}$, $z=-15$, $z=-10$, ..., $z=+15$, $z=+20\text{ cm}$) to the Raytrace Reference Plane at $z=1.0\text{ metre}$, at various momenta ranging from 0.1 GeV/c to 99.0 GeV/c.

- **test_mitray_tmag.F (*.o, executable test_mitray_tmag)**
  This is a simple test program which tests the above package. It is described in further detail in the next section.

Note that they have been modified in the process of optimizing the compilation as well as in the debugging process. The newest version of the relevant files are included in the TMC distribution on the Hermes PC farm: /hermes/pro/tmc/ or /hermes/new/tmc/. As mitray_tmag2.F took too long to compile it was split into a set of smaller files:

- **mitray_tmat_z0.F**
- **mitray_tmat_zm5.F**
- **mitray_tmat_zm10.F**
- **mitray_tmat_zm15.F**
- **mitray_tmat_zm20.F**
- **mitray_tmat_zp5.F**
- **mitray_tmat_zp10.F**
- **mitray_tmat_zp15.F**
- **mitray_tmat_zp20.F**
Table 3: Output parameters from subroutine mitray_tmag_call

<table>
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<tr>
<th>name</th>
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</thead>
<tbody>
<tr>
<td>theta</td>
<td>real*4</td>
<td>polar angle of track at point of nearest approach to beam axis</td>
<td>radians</td>
</tr>
<tr>
<td>phi</td>
<td>real*4</td>
<td>azimuthal angle of track at point of nearest approach to beam axis</td>
<td>radians</td>
</tr>
<tr>
<td>z</td>
<td>real*4</td>
<td>z of point of nearest approach to beam axis; z = 0 corresponds to center of the target; the incident beam travels in the +z direction</td>
<td>cm</td>
</tr>
<tr>
<td>x</td>
<td>real*4</td>
<td>x of point of nearest approach to beam axis; x = horizontal direction, such that (x, y, z) form a right-handed coordinate system</td>
<td>cm</td>
</tr>
<tr>
<td>y</td>
<td>real*4</td>
<td>y of point of nearest approach to beam axis; +y = up in the lab frame</td>
<td>cm</td>
</tr>
<tr>
<td>R</td>
<td>real*4</td>
<td>(\sqrt{x^2 + y^2}), distance of closest approach to the beam axis</td>
<td>cm</td>
</tr>
<tr>
<td>ierr</td>
<td>integer</td>
<td>= 0 if no errors; &gt; 0 if error (e.g., failure to converge)</td>
<td></td>
</tr>
</tbody>
</table>

6 Performance tests

The program test_mitray_tmag reads events from the above MC data file, calls the subroutine mitray_tmag_call to calculate the vertex positions and angles, calculates the differences between the generated and calculated vertex angles and positions, and then displays the results on the screen and/or writes them to a disk file. The user may optionally choose to examine only particles of a specified type, or impose a cut on the generated momentum.

6.1 Timing tests

The UNIX command ‘time’ is used to measure the amount of CPU time required to run a program. We wish to measure only the amount of time used in doing the computation of the vertex angles and positions, and not the amount of CPU time consumed in reading the MC data file. Therefore, in the same directory, we provide a program ‘dummy’ which is identical to the test program ‘test_mitray_tmag’ except that it does not call the subroutine ‘mitray_tmag_call’ to reconstruct the vertex parameters. The difference in the amount of CPU time displayed by the commands ‘time test_mitray_tmag’ and ‘time dummy’ is then a measure of the amount of CPU time consumed in vertex traceback.

The result of these timing tests is as follows. On the computer worf.desy.de (1.4 GHz Pentium-III, LINUX), it takes 0.52 seconds of CPU time to do the vertex traceback for 21721 electron tracks. It therefore takes 23.9 microseconds of CPU time for each track.

6.2 Reliability

The set of tracks in the above-mentioned Monte Carlo data set provides a measure of how frequently the algorithm successfully converges to a consistent set of vertex angles and positions. We select only those tracks where the generated and reconstructed momenta differ by less than 10%; events where the generated and reconstructed momenta do not agree well could indicate tracks which have
undergone secondary interactions after the initial beam-generated event, and are thus suspect. In the Monte Carlo data file, there are 58914 tracks (unweighted) which satisfy this requirement. The algorithm successfully reconstructs the vertex position and angle of 58625 of these, for a success rate of 99.51%. This number is NOT weighted according to Monte Carlo weight.

It is worth examining the success rate for each species of particles separately, in momentum bins. These are presented in Table 4. It is evident that electrons have a higher success rate than hadrons, and within each particle type, higher momentum particles have a higher success rate than lower momentum particles. Again, the numbers in Table 4 are NOT weighted according to Monte Carlo weight.

<table>
<thead>
<tr>
<th>particle</th>
<th>momentum</th>
<th># successes</th>
<th># tracks</th>
<th>success rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron</td>
<td>all</td>
<td>21709</td>
<td>21721</td>
<td>99.94%</td>
</tr>
<tr>
<td></td>
<td>1-2 GeV/c</td>
<td>55</td>
<td>56</td>
<td>98.21%</td>
</tr>
<tr>
<td></td>
<td>2-5 GeV/c</td>
<td>4004</td>
<td>4009</td>
<td>99.88%</td>
</tr>
<tr>
<td></td>
<td>5-10 GeV/c</td>
<td>17596</td>
<td>17601</td>
<td>99.71%</td>
</tr>
<tr>
<td>π⁺</td>
<td>all</td>
<td>15129</td>
<td>15242</td>
<td>99.26%</td>
</tr>
<tr>
<td></td>
<td>1-2 GeV/c</td>
<td>2886</td>
<td>2928</td>
<td>98.57%</td>
</tr>
<tr>
<td></td>
<td>2-5 GeV/c</td>
<td>6961</td>
<td>6995</td>
<td>99.51%</td>
</tr>
<tr>
<td></td>
<td>5-10 GeV/c</td>
<td>4378</td>
<td>4385</td>
<td>99.84%</td>
</tr>
<tr>
<td>π⁻</td>
<td>all</td>
<td>11277</td>
<td>11351</td>
<td>99.35%</td>
</tr>
<tr>
<td></td>
<td>1-2 GeV/c</td>
<td>2130</td>
<td>2157</td>
<td>98.75%</td>
</tr>
<tr>
<td></td>
<td>2-5 GeV/c</td>
<td>5285</td>
<td>5306</td>
<td>99.60%</td>
</tr>
<tr>
<td></td>
<td>5-10 GeV/c</td>
<td>3190</td>
<td>3195</td>
<td>99.84%</td>
</tr>
<tr>
<td>K⁺</td>
<td>all</td>
<td>1570</td>
<td>1573</td>
<td>99.81%</td>
</tr>
<tr>
<td></td>
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<td>138</td>
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<td></td>
<td>2-5 GeV/c</td>
<td>744</td>
<td>747</td>
<td>99.60%</td>
</tr>
<tr>
<td></td>
<td>5-10 GeV/c</td>
<td>699</td>
<td>699</td>
<td>100%</td>
</tr>
<tr>
<td>K⁻</td>
<td>all</td>
<td>988</td>
<td>992</td>
<td>99.60%</td>
</tr>
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<td></td>
<td>1-2 GeV/c</td>
<td>101</td>
<td>104</td>
<td>97.12%</td>
</tr>
<tr>
<td></td>
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<td>99.80%</td>
</tr>
<tr>
<td></td>
<td>5-10 GeV/c</td>
<td>373</td>
<td>373</td>
<td>100%</td>
</tr>
<tr>
<td>protons</td>
<td>all</td>
<td>7952</td>
<td>8035</td>
<td>98.97%</td>
</tr>
<tr>
<td></td>
<td>1-2 GeV/c</td>
<td>2276</td>
<td>2311</td>
<td>98.49%</td>
</tr>
<tr>
<td></td>
<td>2-5 GeV/c</td>
<td>4431</td>
<td>4462</td>
<td>99.31%</td>
</tr>
<tr>
<td></td>
<td>5-10 GeV/c</td>
<td>1026</td>
<td>1028</td>
<td>99.81%</td>
</tr>
</tbody>
</table>

6.3 Accuracy

In Table 5, we display the average (weighted) and RMS (weighted) differences between the generated and reconstructed values of the track’s polar angle $\theta$ [mrad], azimuthal angle $\phi$ [mrad], and longitudinal position $z$ [cm]. It is evident that the accuracy of the reconstruction improves with increasing particle momentum. This can be easily understood, because the transverse magnet affects the tracks of higher momentum less.
<table>
<thead>
<tr>
<th>particle</th>
<th>momentum</th>
<th>$\Delta \theta_{av}$</th>
<th>$\Delta \theta_{rms}$</th>
<th>$\Delta \phi_{av}$</th>
<th>$\Delta \phi_{rms}$</th>
<th>$\Delta z_{av}$</th>
<th>$\Delta z_{rms}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GeV/c</td>
<td>mr</td>
<td>mr</td>
<td>mr</td>
<td>mr</td>
<td>cm</td>
<td>cm</td>
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<tr>
<td>electrons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>all</td>
<td>0.75</td>
<td>1.14</td>
<td>10.40</td>
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<td>2.63</td>
<td>3.65</td>
<td>37.81</td>
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<td>2.93</td>
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<td>2.43</td>
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<td>$\pi^+$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>all</td>
<td>1.69</td>
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</tr>
<tr>
<td>2-5</td>
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<td>2.63</td>
<td>12.58</td>
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<td>1.77</td>
<td>2.99</td>
<td></td>
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<td>5-10</td>
<td>0.70</td>
<td>1.21</td>
<td>7.15</td>
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<td>1.22</td>
<td>2.09</td>
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</tr>
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<td>$\pi^-$</td>
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<td></td>
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</tr>
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<td>1.73</td>
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<td>$K^+$</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>1.26</td>
<td>3.26</td>
<td>12.03</td>
<td>24.04</td>
<td>1.70</td>
<td>3.01</td>
<td></td>
</tr>
<tr>
<td>1-2</td>
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<td>4.31</td>
<td>29.79</td>
<td>48.22</td>
<td>3.09</td>
<td>4.82</td>
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<td>1.81</td>
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<tr>
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<td>1.71</td>
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<td>34.81</td>
<td>2.83</td>
<td>3.90</td>
<td></td>
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<td>18.23</td>
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<td>1.69</td>
<td></td>
</tr>
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<td>3.09</td>
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<td>7.89</td>
<td>16.49</td>
<td>1.27</td>
<td>2.47</td>
<td></td>
</tr>
</tbody>
</table>

7 HAZEX - Using Reference Tracks for Correcting the Vertex Kinematics

A different approach is used in the second reconstruction method, which is implemented in the subroutine HAZEX. Here corrections on the measured tracks are applied based on reference tracks from a database. Those particle trajectories were calculated by means of the ZGOUBI program [2]. This algorithm was written by Witold Augustyniak at the Soltan Institute and is referred to as either "correction method 1" or "Witold’s method".

7.1 Description of the Method

In Fig. 7 the HERMES spectrometer geometry [3] together with the magnetic field map used in the design phase of this program is shown. The field is represented by contours of equal strength in the median plane, i.e., the plane defined by the $z$-axis and the $x$-axis. The reference system used has the $z$-axis along the beam with the $y$-axis directed upwards and the $x$-axis lying in the horizontal plane to form a right-handed coordinate system.\(^3\)

As the vertical acceptance of the HERMES spectrometer extends from 40 mrad to 140 mrad

\(^3\)Note that this differs from the coordinate system used internally in hazex as will be described later.
Figure 7: Extrapolated field map from first measurement along z-axis.

the off mid-plane effects do not play a significant role. The map uses the originally measured values of the field on the z-axis and extrapolated values on the x-axis. The extrapolation has been carried out under the assumption of equal spatial field slopes in both directions.

As mentioned before this map was used at the time when the procedure was being worked on. The same map served also for tests of the method. Comparing the map presented in Fig. 7 with the one calculated by the MAFIA code and subsequently adjusted to the measured values along the z-axis (Fig. 2, see also section 2) one can see that differences in the registered hits in the DVC and the FC2 chambers are very small. One should also stress that it is important that the method works with different maps. In section 7.5 we will discuss the handling of real data. For this case the map should be as close to reality as possible. In general the map should be treated as an important input parameter.

In this method the field map is used to calculate a base set of trajectories. The calculation is done in a separate code 'barka' which uses the program 'Zgoubi' [2]. In the correction procedure the base file will be opened as an direct access file. Looking at the maps one can see that in the region of the DVC and FC2 chambers the magnetic field is very low and the particle trajectories in this region can be approximated with good accuracy by straight lines. The base file contains the indexed records with information from the calculation of the particle trajectories in the magnetic field. The coordinates that are important for the correction are the interception points of the particle trajectory with the DVC and the FC2 chambers and characteristics of the particles i.e. momentum and charge.

In the calculation of the base file the four-dimensional Hermes spectrometer acceptance space has been covered by a regular net of trajectories. Several base files for different maps have been prepared. The base files for the map presented in Fig. 7 and Fig. 2 have been calculated in the
same ranges:

\( p - \text{momentum} \) range \( 0.5 \div 10 \text{ GeV/c} \) with a step of \( 0.05 \text{ GeV/c} \),

\( z - \text{vertex} \) range \( -20 \div 20 \text{ cm} \) with a step of \( 2 \text{ cm} \),

\( \vartheta_y - \text{vertical angle} \) range \( 40 \div 140 \text{ mrad} \) with a step of \( 20 \text{ mrad} \),

\( \vartheta_x - \text{horizontal angle} \) range \( -270 \div 210 \text{ mrad} \) with a step of \( 20 \text{ mrad} \).

For the map presented in Fig. 7 a bigger (extended) file has been prepared with the following ranges:

\( p - \text{momentum} \) range \( 0.5 \div 27.5 \text{ GeV/c} \) with a step of \( 0.05 \text{ GeV/c} \),

\( z - \text{vertex} \) range \( -20 \div 20 \text{ cm} \) with a step of \( 2 \text{ cm} \),

\( \vartheta_y - \text{vertical angle} \) range \( 40 \div 160 \text{ mrad} \) with a step of \( 20 \text{ mrad} \),

\( \vartheta_x - \text{horizontal angle} \) range \( -250 \div 230 \text{ mrad} \) with a step of \( 20 \text{ mrad} \).

The sequence of records in the base file is ordered in such a way that the index of the record denotes momentum, then \( z \) vertex, vertical angle and horizontal angle. In this way the position in the base file of a particular base trajectory can be easily calculated. The sequence of change is reverse to the position in the records, e.g., first 25 records have the same momenta, \( z \)-vertices and vertical angles, but different horizontal angles.

Besides the field map the reconstructed (front-) track parameters of each individual particle track serve as input for the reconstruction. More specifically the momentum of the particle as well as the reconstructed positions on the DVC and FC2 will be used as an input. In the base file, a relation exists between two sets: the momentum, \( z \)-vertex and the vertical and horizontal angles of the trajectory, and the positions on the DVC and FC2 chambers. It is a simple mapping of the first set onto the second set. Unfortunately, it is not a one-to-one relation. Therefore, a simple procedure to look for the correspondent trajectory in the base contains the condition of minimal combined distance \( d \) between interception points of trajectories on the DVC and FC2 chambers:

\[
 d(i, k) = \sqrt{d_1^2(i, k) + d_2^2(i, k)}, \tag{23}
\]

where: \( d(i, k) \) is a sum of combined distances on both chamber, \( k \) runs over the base index of the record, \( i \) is a identifier of the experimental signals from the chambers, \( d_1(i, k) \) is the distance between trajectory positions at the DVC chamber and \( d_2(i, k) \), is the distance between trajectory positions at FC2. This method has the following drawbacks:

1. procedure is ambiguous
2. it is time consuming
3. has low accuracy

These drawbacks ask for another solution. Point 1 suggests to select a subset from the base for which ambiguity will be eliminated. Point 3 suggests to use different characteristics of trajectories instead of simply reading the \( z \)-vertex and the angles. Therefore the vertical angle and \( z \)-vertex will be determined first. In the calculation of the horizontal angle these values will then be used.

The \( z \)-vertex and the vertical angle are determined in a similar way, presented in Fig. 8. The relations written in Fig. 8 as well as in the following description for the \( z \)-vertex are the same also
for the vertical angle:
The real trajectory is shown as a solid line. This trajectory intercepts chambers DVC and FC2 at
certain points. If we assume that in the lowest order the trajectory of the particle is a straight line
(dashed line), the $z$-vertex and $\theta_y$ may be calculated and called $Z_{\text{cal.}}^{Ex}$. Superscript Ex denotes the
experiment and the subscript cal means calculation with linear approximation. The true value is
not known. The difference between the true and calculated vertices may be formally written as
$\delta_0^{Ex}$, such that

$$Z_{\text{true}}^{Ex} = Z_{\text{cal.}}^{Ex} + \delta_0^{Ex},$$

(24)

In the next step we search in the subset of the base for the trajectory which will have the
smallest distance $d(i, k)$, as measured on both the DVC and the FC2 chambers, to the given track.
This subset of the base is defined in the following way:

- momentum from the main magnet analysis is known
- the vertex and the vertical angle calculated in the linear approximation $Z_{\text{cal.}}^{Ex}$ and $\theta_y^{cal.}$.

Let us define a subset in the base limited to the two nearest neighbors of momentum and $\theta_y^{cal.}$ as
well as only one nearest neighbor in the case of $Z_{\text{cal.}}^{Ex}$. When we found the trajectory in the subset
of the base, the difference $\delta B$ between the real and the calculated (using the linear approximation)
vertex is calculated:

$$Z_{\text{true}}^{B} = Z_{\text{cal.}}^{B} + \delta B,$$

(25)

where index B indicates the base subset. The same is done for the vertical angle. These differences
will be used to determine the true values of the $Z$ vertex and vertical angle of the measured track.

---

4. Since the main component of the magnetic field is in the $y$-direction this is more than justified.
This is one key point (also written in the left bottom corner of Fig. 8) of the correction procedure. In this method the base is not dense in order to simply read vertex and vertical angle but we assume that the difference between the linear approximation and the real value are the same for measured tracks and reference trajectories in the base, in other words:

\[ \delta_{0}^{E, x} = \delta_{1}^{B}. \]  \hspace{1cm} (26)

Figure 9: Dependence of the horizontal angle on the track position at the FC2 chamber for two different momenta and fixed \( z \)-vertex and vertical angle

After the \( z \)-vertex and the vertical angle have been determined the calculation of the horizontal angle is very simple. It is illustrated in Fig. 9, which shows the relation between the horizontal angle and the position at the FC2 chamber for fixed momenta, given \( z \)-vertex, and vertical angle. One can see that this simple relation can be fitted easily by a polynomial of lower orders. From the fit the horizontal angles for fixed momenta, fixed vertical angle and fixed \( z \)-vertex is determined. Hence the problem is reduced to the interpolation of horizontal angles depending on the momentum determined by main magnet and the vertical angle and \( z \)-vertex calculated as described earlier.

At the end of this section the spectra of the error for the determined vertices, horizontal and vertical angles are presented in order to get a feeling what kind of uncertainty is coming from the method alone. For this tracks have been generated that were not subject to any detector smearing, i.e., they were only traced through the magnetic field. In real life one has to also consider the influence of detector smearing and inaccurate reconstruction of the front track parameters. This will be discussed in section 7.3. Therefore, here only the errors due to the method are considered and are shown in Fig. 10. The base file used for this analysis was constructed from the map presented in Fig. 7. In addition, Figs. 11-13 show the minima and maxima for the corrections of \( \theta_x \), \( \theta_y \), and \( z_{\text{vtx}} \). As expected that largest corrections are for \( \theta_x \), reaching values of up to 100 mrad.
Figure 10: The errors for the method for both the vertical and horizontal angle as well as for the z-vertex for tracks with momentum between 1.5 and 2.5 GeV.

Figure 11: The minimum (circles) and maximum (squares) shifts of the horizontal angle in mrad due to the target magnet, as a function of the momentum of the particle. Shifts are in opposite directions for positive vs. negative particles.
Figure 12: The minimum and maximum shifts of the vertical angle in mrad as a function of the momentum of the particle.

Figure 13: The minimum (circles) and maximum (squares) shifts of the vertex position in cm as a function of the momentum of the particle.
7.2 A few more comments about the correction

The idea behind this correction procedure is very simple. The main routine hazex may be easily implemented in other codes as well. Actually the importance lies in the base file. It is opened as a direct access file and the address of the record is calculated by the function num with following arguments: momentum, z-vertex, vertical and horizontal angle. The simple programs hata1.f and hata2.f\(^5\) can be used to check the base set. The version of num in the correction program should be the same as the one in the simple code.

The program takes advantage of the HERMES spectrometer symmetry\(^6\), therefore the base file is defined only for positive values of the vertical angle and the particle charge. All other cases can be transformed back to this case. If, e.g., the \(y\)-positions at the DVC and FC2 chambers are negative one just needs to change these values into positive ones, calculate the positive value of the vertical angle and its correction and apply it (remembering the sign) to the original negative value of the vertical angle. The determination of the horizontal angle is not affected by this as isn’t the \(z\)-vertex calculation.

When one has negative particles it will, however, influence the horizontal angle. Nevertheless it is a simple procedure because it only changes the sign of the horizontal angle leaving the vertical angle and again the \(z\)-vertex untouched. In praxis this is achieved by reversing the sign of the horizontal chamber positions.

As mentioned already before the base file uses the opposite definition of the \(z\)-direction from the one HERMES normally uses, i.e., the farthest point of the target from the spectrometer magnet is at \(+20\)cm whereas the closest point is at \(-20\)cm. This should be kept in mind when using the base file.

Last but not least the base file is defined for a beam in the median plane. However, a vertical shift can be introduced and considered in the reconstruction. Small shifts can be treated by just adding this value (internally) to the vertical chamber positions. A procedure to find a possible \(y\)-shift will be described in section 7.4.

At the end of this section we want to look at the distribution of trajectories at the two chambers. As one can see in Fig. 14, all the simulated trajectories coming from all possible points of the target and being accepted by the HERMES spectrometer when running with a transverse target magnet, are grouped in characteristic loci at the DVC and FC2 chamber. In Fig. 14a(b) the relation between the \(x\)(\(y\))-positions are presented. In the analysis particles emitted from points different from the target are treated as background. The \(x\)- and \(y\)-positions may only be inside the contours defined in Fig. 14. For the particles which lay outside these contours, the predicted values for the vertex position and the angles will be unreasonable.

7.3 Performance Check using MC Data

For a performance check the Monte Carlo data from section 3 has been used. The used map of the magnetic field is the measured field map for the years 2003-2005.

The simulated data provide the particle momentum and track positions on the DVC and FC2 chambers with uncertainties similar to those encountered in the experiment. The biggest relative errors in the momentum determination is expected for positrons with low energies.

In this section the \(\sigma\)'s of the Gaussian error distributions as a function of the momentum of selected particles are presented as well as the means of the distributions. These results are shown in Figs 15-17. As one can see the biggest error occurred at lower energy. The mean values for the errors in the horizontal angle in the case of positron are presented in the upper panel of Fig 15.

\(^5\)They are not part of the TMC distribution.
\(^6\)This changes with the new measured field maps available now.
They are very similar to the ones in case of unpolarized (or longitudinally polarized) targets. The $\sigma$’s for the horizontal angle errors at low momenta reach values up to 5 mrad. At higher momenta this value drops below 1 mrad. In the case of vertical angles these values are similar. One might have suspected that the resolution would be better in the vertical angle as in this method the vertical angles depend only on the vertical positions on the DVC and FC2 chambers. In contrast the horizontal angle depends on the momentum, the horizontal positions at the two chambers, the vertical angle and the $z$-vertex which are all arguments in the interpolation procedure and thus contribute to the overall smearing. The resolution in the vertex position as well becomes better with increasing momentum, but not much better than 2cm.

One point which should be stressed again is that the presented results do not differ significantly from the results using HMC without a transverse holding field (and thus without track correction). This shows that the additional uncertainty introduced by the method is small as stated already in the previous section.

7.4 Self-Alignment Method for Vertical Beam Shifts

In this section the errors induced by a beam shift in the vertical direction and also a method how the beam shift may be determined from real data will be considered. Additionally, a way is shown how to reduce such errors when information about $y$-shift are known.

As explained already in order to get the subset of trajectories in the base when looking for the

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7There are additional effects coming from the standard track reconstruction, e.g., the better resolution in the vertical direction because of the 'X-U-V' wire arrangement, the lower limit of 40mrad on the vertical angle and the impact of force bridging on the horizontal angle of the front track.
Figure 15: The width and mean values of Gaussian fits to the difference in the generated and reconstructed $\theta_2$ values for electrons and pions (other particle types behave similar and have been left out for clarity).
Figure 16: The width and mean values of Gaussian fits to the difference in the generated and reconstructed $\theta_y$ values for electrons and pions (other particle types behave similar and have been left out for clarity).

Figure 17: The width of Gaussian fits to the difference in the generated and reconstructed $z_{vtx}$ values for electrons and pions (other particle types behave similar and have been left out for clarity).
trajectory with minimal combined distance on both chambers the $z$-vertex and the vertical angle were calculated in a linear approximation. The vertical angles are defined by the $y$-positions on the DVC and FC2 chambers. A determination of the vertical angle does not depend on the beam position. However, the $z$-vertex is defined by the intercept of the track with the median plane where the beam is supposed to be located. The vertical angles measurable with the HERMES spectrometer are small ($40 \div 140$ mrad). Therefore, a $y$-shift of the beam in the order of a millimeter may create a shift of the reconstructed $z$-vertex in the order of a cm depending on the vertical angle. This observation and a schematic description of the here discussed self-alignment method is presented in Fig. 18.

![Figure 18: Self alignment method for determining $y$-shifts of the beam.](image)

The method should work with data with the transverse magnet switched on and also switched off. It is just reduced to the calculation of the value $Z_{calc.}$ defined in section 7.1. Figure 18 presents an event with two detected particles coming from the same $z$-vertex (for example coming from decay of the $\rho^0$ meson). The tracks are selected in such a way that particles are emitted in opposite vertical directions from the point shifted in $y$ direction. This real vertex is marked as $Z_0$, but the procedure will determine two vertices $Z_1$ and $Z_2$ for the plotted tracks. The $z$-vertex depends on the emitted angles i.e. vertices for particles emitted with positive (negative) vertical angles are shifted to negative (positive) values when the vertical shift of the beam is in the positive $y$-direction.

The relation between the various $z$-vertices, the vertical angles and the vertical beam shift is

$$h = \frac{s}{\cot(\vartheta^z_1) + \cot(\vartheta^z_2)},$$

(27)

where $s$ is the distance between the two $z$-vertices, $\vartheta^z_1$ and $\vartheta^z_2$ are the vertical angles of the two tracks. One can see from Fig. 18 that the particle with index 1 and $\theta_y > 0.0$ will be shifted from the calculated value and for $\theta_y < 0.0$ will also be shifted but to the opposite side. The method
is much more general, e.g., not only $\rho^0$ vector mesons can be used. Above relation was used to determine the $y$-shift in real data.

For a test the data from 1997 was chosen. The usefulness of this simple method depends on the resolution and requires well defined events without background. The spectra for the $y$-shift of the beam defined by Eq. 27 are shown in Fig. 19(a). The well pronounced peak is shifted by -0.016 cm. It is a very small shift but the sign is the same and confirmed in Fig. 19(d), where the difference of the $z$-vertices is shown. Also in this spectrum the peak is shifted to negative values and centered at -0.7 cm. It may be stated that the resolution and the statistics is proper to define a shift like that. In our case the events were selected in the following way:

1. events with $\rho^0$ were selected by a cut on the invariant mass from 0.6 up to 1.1 GeV/c.
2. $\Delta E < 0.6$ and $-t' < 0.4$ to obtain exclusive $\rho^0$ mesons with little background
3. pions from the decaying $\rho^0$ have vertical angles of opposite sign

Figure 19: Determination of the $y$-shift of the beam using 1997 data. a) The determined $y$-shift using $\rho^0$ events. b) The invariant mass of the two pions for the selected events. c) Dependence of the $z$-vertex of pions emitted at negative vertical angles vs. the $z$-vertex of pions emitted at positive vertical angles (both pions coming from selected $\rho^0$ events). d) Difference between the $z$-vertices of the two pions.

The resulting invariant mass spectrum with these conditions is shown in Fig. 19(b).

The vertical shift of the beam was determined with $\sigma = 0.038$ cm, and the difference of the $z$-vertices with $\sigma = 1.4$ cm. The two-dimensional spectrum in Fig. 19 presents the $z$-vertices of the
pions which fulfill conditions 1 and 2. There are plenty events placed far from the diagonal. From our consideration only the events within $2\sigma = 2.8$ cm should be accepted. It is also easy to estimate the difference of $z$-vertices from Eq. 27. The events far from diagonal should thus be treated as a background.

The background can also be determined from Fig. 20. The two dimensional histogram shows the difference of the vertices ($e^+ - \pi^+$) versus the difference ($\pi^+ - \pi^-$). Good events are concentrated in the middle near zero. It is easy to see that there exist events far from the central region.

Figure 20: Distribution of events which are assumed to be $\rho^0$ events. Plotted is the difference of the $z$-vertices of the $e^+$ and the $\pi^+$ vs. the difference for the pion pair. The distribution is – as expected – concentrated around zero but there are tails which might originate from background events.

The same test was done for events with transverse magnet switched on. A set of the events has been simulated using HMC with the beam shifted by 0.3 cm. In order to determine the vertical shift from this MC set, events similar to those considered before were selected: Three-particles events with positrons and two oppositely charged pions. It has been shown before that the errors for particles with higher energies are smaller. Therefore, following conditions on the particle’s momenta were put:

- pions higher than 1 GeV
- positron higher than 5 GeV.

Additional conditions on the invariant mass of the two pions were also introduced. The statistics of the data sample used for the determination of the shift was rather poor. Nevertheless the vertical shift of the beam could be reproduced. The calculated $z$-vertices for this set of events are presented in Fig. 21(a-c). The different spectra show the error in the $z$-vertex for (a) $e^+$, (b) $\pi^+$ and (c) for $\pi^-$. (Panel (d) shows the invariant mass spectrum.) The dashed lines in Fig. 21(a-c) describe the results calculated without information about the shift. The solid lines are the results calculated with considering the determined vertical shift.
Figure 21: Spectra of errors of the reconstructed $z$-vertex (panels a-c), determined from a Monte Carlo simulation which has a $y$-shift of the beam of 3 mm. The solid (dashed) lines are the results with (without) correction of the beam shift in the calculations. The selected data sample consists of 3-track events with the invariant mass of the 2-pion system being in the region of the $\rho^0$ (panel d). The different distributions are a) $e^+$ with momenta bigger than 5 GeV, b) $\pi^+$ with momenta bigger than 1 GeV and c) the same as b) for $\pi^-$. 
The event selection prefers pions with rather low energies. The pions from selected events were then used to determine the vertical shift of the beam. This is shown in Fig. 22. The dashed line represents the spectra of the vertical shift calculated according to Eq. 27. The maximum indicates a shift of 0.3 cm, the true value of the shift. The spectrum has been calculated again including the information about the vertical beam shift in the correction. It has its maximum as well as its mean value near zero.

Figure 22: Determined $y$-shift of the beam using the Monte Carlo data as described in text. Tracks from 3-track events were used with an invariant mass of the 2-pion system around the $\rho^0$ peak (see also Fig. 21). The solid (dashed) line was calculated with (without) $y$-shift correction.

To sum up this section it may be stated that the vertical shift of the beam is important for the analysis and by events like $\rho^0$ events it may be controlled and corrected.

7.5 Real Data

In this concluding section a first look at real data is summarized. This is one study besides the many studies done in other analyses (cf. [7]). It uses data from the first HRC production of transverse-target data, i.e., the 02a production. Events were read using the HEXE tool requiring 3-track events. Those are very useful for the determination of the vertical shift of the beam by the method presented in the previous section.

The momenta and charges were determined by the main magnet. It was also possible to determine the position at the DVC and FC2 as well as to use a crude PID, i.e., select leptons and hadrons. The following conditions were used for leptons:

- $0.8 < E/P < 1.1$
- $16.0 < \text{TRD} < 100.0$
- $0.01 < \text{preshower} < 1000$
In the case of hadrons the following cuts were used:

- $E/p < 0.8$
- $0.1 < \text{TRD} < 14$
- $0.0 < \text{preshower} < 0.0075$

Tracks that did not fulfill those conditions were rejected. It was also assumed that the identified hadrons were pions. In Fig. 23 the particle yields separated in this way are presented. One can see that mainly pions (hadrons) were produced.

![Figure 23: Particle types according to hard cuts on the PID detectors (see text) for the selected three-track events (the LUND particle code is used even though this is real data!)](image)

In Fig. 24 the track distribution at the DVC and the FC2 are shown as discussed already in section 7.2. The events are inside the contours which is another argument for the statement that reconstructed particles were coming from the target.

At last a similar analysis as done for the MC data when looking for vertical shifts has been done. The following cuts were used for the vertices of the particles:

1. $|z_{vtx}| \leq 20 \text{ cm}$ for both pions
2. $|z_{vtx}^1 - z_{vtx}^2| \leq 6 \text{ cm}$ (which corresponds to two $\sigma$ of the spectrum of the difference of the $z$-vertices for tracks coming from the same event. (not shown))

The surviving events are shown in Fig. 25(b). It is the 2-dimensional distribution of both pion vertices.
Figure 24: Position of the particles at the DVC vs. the position at the FC2 for either the $x$ direction (a) or the $y$ direction (b). The lines define the region for particles originating from the beam inside the target cell.

The results are presented in the remaining panels of Fig. 25. In panel (a) the calculated vertices are shown. The particles emitted at positive vertical angles are marked by superscript '1' and their distribution is the solid histogram. Negative vertical angles are marked with superscript '2' and are shown with a dashed line. The shapes are the same but the plot suggests a small shift of the $\pi^1$ spectrum compared to the $\pi^2$ case. The calculated vertical shift of the beam also shows a very small shift of -0.017 cm (panel (c)) but the determined $\sigma$ for this spectra is one order of magnitude higher. Therefore, the invariant mass of two pions were calculated without introducing a $y$-shift in the correction and is shown in panel (d). One can nicely see the $\rho^0$ peak.

8 TMC

The main purpose of TMC is to provide a program which embeds the two described correction procedures in a framework which is DAD [5] compliant and thus can be used in the HERMES data production chain. An easy way to do so was to use the HANNA [6] frame. For this particular purpose it should be able to read HRC files. As all subroutines are already written in FORTRAN77 also TMC is written in FORTRAN77. In this section some details about the structure of the program are given, in particular, some of the various command-line options are described.

All the calls to the correction routines are done in HANNA’s user_event function. This is done for each single track. The rcPartTrack and rcTrack tables are read from the hrc.devents file to get the information about the reconstructed track. The track location at the DVC and FC2 are calculated using the front partial track information, the momentum and charge come from the rcTrack table. In case of Monte Carlo data, the option is provided to also read in the generated track parameters for a quick comparison. This option can for instance be used in conjunction with the tmcdebug mode where both generated and reconstructed track parameter are printed into
additional output files.

But before user_event is called there are a few more things which have been done. For Witold’s correction method the base files has to be opened (done in user_init). Also in user_init all the command line arguments are checked and the DAD output file is created. In run_init it is checked that the current run number actually is in the list of runs with target magnet on. For this it is at the moment still required to provide a runlist if one wants to use this option.

Depending on the chosen correction method the various correction routines have to be called. Error codes are provided depending on the responses of the correction routines. They are stored together with the corrected vertex values and a correction method identifier in the new rcVertexCorr table. When a method failed to find corrected values, -999 is stored instead. All other tables of the hrc.devents file are copied over without change.

One comment is at place about what is stored in the fields for the helix parametrization. While Stan’s method provides a reconstructed particle’s trajectory in the target region that is not required to coincide with the beam trajectory at some point, i.e., that does not necessarily reaches a coordinate of the form \((0, 0, z)\), Witold’s method always gives a trajectory that reaches the \((0, 0, z)\) line. Hence, in the latter case giving a helix parametrization is not necessary as it can easily be derived from the values for the \(z\)-vertex positions as well as the azimuthal and polar angles of the track at that point. The disadvantage of this is that for particles that do not come from the beam line, e.g., decay particles from longer-living particles, this information is to be used with care as it hardly gives the right answer due to the prerequisite of the method of having a track from the beam line. In contrast, Stan’s method does provide additional information as the reconstructed
vertex kinematics are the ones of the transported particle-trajectory’s closest approach to the beam line. Internally, this trajectory is already treated as a helix, so it can be used to find the closest approach to any line or curve of interest. The stored helix parameters are the ones needed for the parametrization \((x(z), y(z), z)\), where \((x(z), y(z))\) are defined in Eqs. (16) and (20)\(^8\) (see also Appendix B). In addition, the coordinates \((x, y, z)\) of the calculated closest approach of the helix to \((0, 0, z)\), i.e., \((x(z_{vtx}), y(z_{vtx}))\), and also the track’s horizontal and vertical angles \(\theta_x\) and \(\theta_y\) are given partially for convenience as some of them can be calculated from the helix parametrization.

In order to correct for possible misalignment effects, the possibility is given to adjust the track positions at the FC and DVC locations. This can be done via the command-line arguments \texttt{misalignXtop} and similar for \(y\) and bottom. These values (in cm) will be subtracted from the track’s \(x\)- and \(y\)-positions at the FC and DVC as determined by HRC. In principle, only detector shifts can be corrected by such a method as a shifted beam would see a slightly changed magnetic field. However, for small shifts, detector and beam shifts should be equivalent.\(^9\)

One peculiarity to Witold’s correction method is the exploitation of symmetry aspects at HERMES. Therefore the input data to hazex is first checked for the particle charge and the sign of the vertical angle of the track. Depending on their values the track positions at the DVC and FC2 are changed accordingly (cf. section 7.1) and this transformation is saved for later consideration.

As by now various field maps and thus also various base files are available some modifications to hazex have been done to have one routine for all field maps. Actually in TMC there are 2 versions in use: the original for the 02a1 field map (cf. appendix A) and one version for all subsequent field maps. The difference between the two originates from the nature of the base files: all newer base files use integer values which changes how the information stored in the base file is treated by hazex. Newer base files are also available for the two charges separately.

References


\(^8\)In Eq. (20) \(\phi\) denotes the vertical angle \(\theta_y\)

\(^9\)Presently it is even impossible to precisely determine the position of the beam with respect to the target magnet.
A Command Line Arguments

A list of command line arguments can be obtained via TMC --help.\(^\text{10}\)

- **mc** uses information from the Monte Carlo tables to provide a quick comparison of generated and
corrected track parameter (useful with -debug option)

- **debug** will run in debug mode and provide additional output ASCII files

- **--HSG** run on HSG instead of (the default) HRC files

- **--method** choice of correction method to use: 0-none Default:1-Witold 2-Stan 3-both

- **--fieldversion** version of the used field map (default: 02a1), presently following field maps are
available:
  - 02a1: scaled and stretched version of the MAFIA calculation; for Witold’s method the
    base contains trajectories for 0.5<\(p\)<10GeV
  - 02a2: as before but 0.5<\(p\)<27.5GeV
  - 02b1: 2002/03 measured map (extrapolated)
  - 03a1: 2003+ measured map (extrapolated); 0.5<\(p\)<10GeV
  - 03a2: 2003+ measured map (extrapolated); 0.5<\(p\)<27.5GeV

- **--pmin** lowest momentum for which correction will be attempted (default: 0.3 GeV). Note that
  for Witold’s method only particle with momenta bigger than 0.5 GeV will be reconstructed
  reliably, while Stan’s method should work down to 0.1GeV.

- **--misalignXtop** \(x\)-offset of beam as seen by HRC (HERMES coordinates) (default: 0.0 cm)

- **--misalignXbot** \(x\)-offset of beam (default: 0.0 cm)

- **--misalignYtop** \(y\)-offset of beam (default: 0.0 cm)

- **--misalignYbot** \(y\)-offset of beam (default: 0.0 cm)

- **--odriver** ADAMO driver for output (WFIL (default), WPIP, WMEM)

- **--ofile** \(<\text{filename}>\) name of output file (default: tmc.devents)

- **--TMagRunlist** \(<\text{filename}>\) uses runlist \(<\text{filename}>\) to determine status of target magnet for
  the various runs; if none is given every run will be corrected (except for --method 0) (default:
  NONE)

- **--version** shows version and some release information

B The rcVertexCorr Table

All information in the original hrc.devents file are kept. The information from the correction is
stored in the new table rcVertexCorr (to which a link points from the rcTrack table) which is
defined in the HRC.ddl:

**Theta:** corrected \(\theta\) for the track \([\text{rad}]\)

\(^{10}\)This list is up-to-date as of version 10.0 of TMC.
**Phi:** corrected φ for the track [rad]

**ZVx:** corrected z-vertex position of the track [cm]

**HelixPar:** parameter defining the helix track inside the target cell, i.e.,

\[
\begin{align*}
x(z) &= x_0 - \hat{q}\sqrt{R^2 - (z - z_0)^2} \\
y(z) &= y_i + (z - z_i) \tan \theta_{y,i}
\end{align*}
\]

and some additional information. In particular, HelixPar\[*]= [\(x_i, y_i, z_i, \theta_{x,i}, \theta_{y,i}, x_0, z_0, R]\],

where

\((x_i, y_i, z_i)\): are some initial track coordinates, usually the reconstructed vertex point [cm],

\(\theta_{x,i}, \theta_{y,i}\): are the horizontal and vertical angles at \((x_i, y_i, z_i)\) [rad],

\((x_0, z_0)\): is center of the circular arc in the x-z-plane [cm], and

\(R\): is the radius of the circle [cm].

**Method:** correction method used (0=none, 1=Witold, 2=Stan)

**Status:** error code from the correction routines, in general < 0 = error, 0 = o.k., > 0 = tracks were outside scope of base set (Witold’s method) or could be tracked to the target reference plane but TMC failed to find closest approach to \((0, 0, z)\) (Stan’s method), but these tracks were corrected anyway, e.g., following bits can be set in that case for Witold’s method:

- **0x00000001:** \(p < 0.5\)GeV. TMC used 0.5GeV for correction and extrapolated,
- **0x00000002:** \(p < 27.5\)GeV. TMC used 27.5GeV for correction and extrapolated,
- **0x00000004:** \(\theta_{x}^{HRC} < -220\)mrad. TMC used -220mrad for correction and extrapolated,
- **0x00000008:** \(\theta_{x}^{HRC} > 220\)mrad. TMC used 220mrad for correction and extrapolated,
- **0x00000010:** |\(\theta_{y}^{HRC}\)| < 40mrad. TMC used 40mrad for correction and extrapolated,
- **0x00000020:** |\(\theta_{y}^{HRC}\)| > 140mrad. TMC used 140mrad for correction and extrapolated,
- **0x00000040:** \(z_{vtx}^{HRC} > 20\)cm. TMC used 20cm for correction and extrapolated,
- **0x00000080:** \(z_{vtx}^{HRC} < -20\)cm. TMC used -20cm for correction and extrapolated;

while following status information can occur for Stan’s method:

- **1:** \(z\) of y-intersect (see Eq. (29)) is outside of z-range covered by circle (28),
- **2:** no intersect of circular arc (28) with \((0, 0, z)\) exists for \(z\) within ±25cm,
- **3:** although intersect should exist, could not converge to a value after 30 iterations.

Especially in the case of Stan’s method, a positive status value could come from tracks that don’t originate from the beam line inside the target, e.g., collimator scattering or decay particles from, e.g., hyperons, but could also indicate a wrong assumption of where the beam is located.
C  The Measured Field Maps

There are several field maps available which come from measurements of the target magnet. Originally, only a measurement along the $z$-axis existed which were used in conjunction with a MAFIA simulation of the target magnet to get a three-dimensional field map. This map is referred to as the 02a map.

During the Spring 2003 shutdown, the target magnet has again been surveyed leading to an extended measurement of the magnetic field. It was also modified to provide a good homogeneity inside the target cell despite of the additional magnetic shielding needed for the functioning of the lambda wheels downstream of the target. The file transfield2002.map contains the measured field map before the addition of magnetic shielding to shield the lambda wheels. It pertains to all transversity data taken before March 2003. The measured data points do not span the entire region spanned by the particles within the spectrometer acceptance. The deficiency is especially serious in the vertical direction, since the measured data points span only the region from $y = -4$ cm to $y = +6$ cm. In the initial field map, the sign of the $B_x$ component is also wrong.

These deficiencies are corrected in the file transfield2002.extended.map. Here, additional data points are added by taking each vertical ($y$) slice, and extrapolating from the measured points in the region $y = [-4, +6]$ cm out to $y = [-10, +10]$ cm. Third order polynomials, whose coefficients were fit to the measured data points, were used to extrapolate the $B_x$ and $B_z$ components, and 4th order polynomials were used to extrapolate the $B_y$ components. The sign error in $B_x$ is also corrected. Although the extrapolation is carried out to $y = [-10, +10]$ cm in order to avoid computational artifacts arising from interpolating from points right at the edge of the map, it is deemed that the extrapolated field is not accurate past $y = [-8, +8]$ cm. This is the field map which is used when fieldversion 02b1 is specified (see Appendix A).

It should be noted that there are no measurements out in the extrapolated region, and thus it is highly uncertain how accurate the extrapolation really is. The extension of the field map by extrapolation is driven by necessity; we made a reasonable guess based on a smooth extrapolation from the measured data points, but it really is only a guess not verified by any measurements.

The field transfield2003.map contains the measured field map after the addition of magnetic shielding to shield the lambda wheels, and pertains to all transversity data taken after May 2003. The sign in $B_x$ is also wrong. The additional shielding further constricts the region that could be measured, and the mapped region covers only $x = [-10, +8]$ cm horizontally and $y = [-4, +6]$ cm vertically. The measured region is too small in both the horizontal and vertical directions. The map was extended, first by extending horizontally out to $x = [-14, +14]$ cm in the relevant region, to accomodate the rays near the edge of the horizontal acceptance. Then each vertical slice is extended from $y = [-4, +6]$ cm to $y = [-12, +12]$ cm, using extrapolating polynomials of the same order as described above for the “2002” map. Again, it must be emphasized that the accuracy of the field values in the extrapolated region is unknown because there are no measurements there against which to check the extrapolation. The file transfield2003.extended.map contains the extended field map, with the sign of $B_x$ corrected. This is the field map which is used when fieldversion 03a1 and 03a2 are specified (see Appendix A).

For Witold’s method there are usually more than one base file per field map as (for space reasons) in addition to a base file that covers the whole momentum range of 0.5GeV to 27.5GeV a second base file that covers only momenta up to 10GeV. Above that the corrections should anyway be tiny. The second base file leads to a subclassification of the -fieldversion command-line argument as described in Appendix A. For the standard productions, the base file covering the whole momentum range is used.