Study of Spin Density Matrix Elements in hard
exclusive Electroproduction of $\phi$ meson on
Proton and Deuteron at HERMES

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Abstract. Exclusive electroproduction of $\phi$-mesons is studied by scattering longitudinally polarized positrons(electrons) off unpolarized hydrogen and deuterium targets in the kinematic region $\langle Q^2 \rangle = 1.9$ GeV$^2$, $\langle W \rangle = 4.8$ GeV and $\langle -t' \rangle = 0.13$ GeV$^2$. Twenty three spin density matrix elements are measured, including for the first time those sensitive to beam polarization. The s-channel helicity is found to be conserved in the transition of virtual photon to vector meson.

Keywords: vector meson, helicity amplitude, SDME, SCHC

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SPIN DENSITY MATRICES AND AMPLITUDES IN THE
REACTION $e + N \rightarrow e' + \phi + N$

The study of exclusive electroproduction of vector mesons is a wealthy source of information on the production mechanism and structure of the nucleon. The process of electroproduction of the vector meson is factorized into two consecutive processes. The incident lepton radiates a virtual photon which dissociates into a $q\bar{q}$ pair. This pair lasts long enough to interact strongly with the nucleon and to form the observed vector meson.

The angular distribution in exclusive electroproduction of vector mesons depends on Spin Density Matrix elements involving the spin state of the virtual photon and the vector meson. The first subprocess of vector meson production, the emission of a virtual photon ($e \rightarrow e' + \gamma^*$), is described by the photon spin density matrix \[1\]

$$\rho^{U+L}_\lambda(e, \Phi) = \rho^{U\lambda'}_\lambda + P_{beam}\rho^{L\lambda'}_\lambda,$$

where U and L denote an unpolarized and polarized beam, $e$ is the ratio of fluxes of longitudinal and transverse virtual photons, $\Phi$ is the angle between the lepton scattering and the hadron production planes. This spin density matrix can be calculated from QED. The vector meson spin density matrix $\rho_{\lambda_V\lambda_V'}$ is expressed by the helicity amplitudes $F_{\lambda_V,\lambda_V'}(W, Q^2, t')$. These amplitudes describe the transition of the virtual photon with helicity $\lambda_\gamma$ to the vector meson with helicity $\lambda_V$ where $\lambda_N, \lambda'_N$ are the helicities of the nucleon in the initial and final states, respectively. In the CM frame of $\gamma^*N$, the spin
density matrix is given by the von Neumann formula [1]:

\[ \rho_{\lambda v, \lambda' v'} = \frac{1}{2\mathcal{N}} \sum_{\lambda T \lambda' T'} F_{\lambda T; \lambda' T'}^{U+L} F^*_{\lambda' T'; \lambda T} \rho_{\lambda' T'}^{U+L} F_{\lambda T; \lambda' T'}^{U+L} \rho_{\lambda T}^{U+L}. \]

(2)

After the decomposition of \( \rho_{\lambda v, \lambda' v'}^{U+L} \) into the set of nine Hermitian matrices \((3 \times 3) \Sigma^\alpha\), the spin density matrix becomes dependent on \( \alpha \): \( \rho_{\lambda v, \lambda' v'}^{\alpha} \), with \( \alpha = 0, \pm 3 \) - corresponding to various polarization states of a transversally polarized photon; \( \alpha = 4 \) - to a longitudinally polarized photon; \( \alpha = 5, 6, 7, 8 \) - to interference terms. When we cannot separate transverse and longitudinal photons, the Spin Density Matrix Elements (SDMEs) are defined as

\[ r^{04}_{\lambda v, \lambda' v'} = (\rho^0_{\lambda v, \lambda' v'} + \varepsilon R \rho^4_{\lambda v, \lambda' v'})/(1 + \varepsilon R), \]

(3)

\[ r^{\alpha}_{\lambda v, \lambda' v'} = \frac{\rho^\alpha_{\lambda v, \lambda' v'}}{(1 + \varepsilon R)}, \alpha = 1, 2, 3, \frac{\sqrt{R} \rho^\alpha_{\lambda v, \lambda' v'}}{(1 + \varepsilon R)}, \alpha = 5, 6, 7, 8. \]

(4)

where \( R = \sigma_L/\sigma_T \) is the longitudinal-to-transverse cross section ratio.

 Usually, the helicity amplitude is decomposed into the sum of an amplitude \( T \) for natural-parity exchange (NPE) \((P = (-1)^J)\) and an amplitude \( U \) for unnatural-parity exchange (UPE) \((P = (-1)^J)\), given by \( F_{\lambda v, \lambda' v'; \lambda T; \lambda' T'} = T_{\lambda v, \lambda' v'} + U_{\lambda v, \lambda' v'} \).

For an unpolarized target there is no interference between NPE and UPE amplitudes and there is no linear contribution of nucleon-helicity-flip amplitudes to SDMEs (as they are suppressed by a factor \( (\alpha)^2 = (\langle \lambda - \lambda' \rangle)^2 \), with \( \lambda' = t - t_{\text{min}} \)). This reduces the number of NPE amplitudes to five: the helicity conserving \( T_{00}, T_{11} \), and the helicity non-conserving \( T_{01}, T_{10}, T_{1-1} \), where we used an abbreviation \( T_{\lambda v, \lambda' v'} = T_{\lambda v, \lambda' v'}^{\pm 1, \pm 1} \). The dominance of diagonal transitions is called s-channel helicity conservation (SCHC).

For a longitudinally polarized beam and unpolarized target there are 23 SDMEs; 15 are unpolarized which do not depend on beam polarization, and 8 are polarized which depend on beam polarization. They are determined from a fit of the angular distribution of kaons in the decay \( \phi \Rightarrow K^+K^- \).

**RESULTS**

The extracted SDMEs of \( \phi \) meson production for the integrated data \( \langle Q^2 \rangle = 1.9 \text{ GeV}^2 \), \( \langle W \rangle = 4.8 \text{ GeV} \) and \( \langle -t' \rangle = 0.13 \text{ GeV}^2 \) are presented in Fig. 1. The SDMEs of the \( \phi \) meson for the hydrogen and deuterium data are found to agree within their statistical uncertainties, so the SDMEs are presented for the combined hydrogen and deuterium data sets (full circle). For comparison, the SDMEs of \( \rho \) mesons for the hydrogen are also shown (full squares). The 8 polarized SDMEs are presented in the shaded areas. Their experimental uncertainties, for both mesons, are larger in comparison to unpolarized
SDMEs because the lepton beam polarization is smaller than unity, and in the equation for the angular distribution they are multiplied by a small kinematical factor [2].

In Fig. 1 the SDMEs are shown multiplied by certain numerical factors in order to allow their comparison at the level of dominant amplitudes. The presentation of extracted SDMEs is based on the hierarchy of Natural Parity Exchange helicity amplitudes (NPE) [2]. In such a hierarchy SDMEs are categorized into the following five classes: Class A corresponds to the transition of longitudinal virtual photon to longitudinal mesons $\gamma_L^* \rightarrow V_L^0$ and transverse virtual photon to transverse mesons $\gamma_T^* \rightarrow V_T^0$. For this class, $|T_{11}|^2 \approx 1 - r_{00}^{04} \approx r_{1}^{-1} \approx -Im\{r_{1}^{2}\}$. Class B corresponds to interference of these two transition and $Re\{T_{00}T_{11}^*\} \approx Re\{r_{10}^{04}\} \approx -Im\{r_{10}^{5}\}$. Class C contains SDMEs which correspond to transitions $\lambda_V \neq \lambda_T$ and the dominant amplitude terms are: $Re\{T_{11}T_{01}^*\} \approx Re\{r_{10}^{04}\} \approx Re\{r_{10}^{1}\} \approx Im\{r_{10}^{2}\}$, $Re\{T_{01}T_{20}^*\} \approx r_{10}^{5}$.
$|T_{01}|^2 \approx r_{00}^1, \text{Im}\{T_{01}T_{11}^*\} \approx \text{Im}\{r_{10}^3\}, \text{Im}\{T_{01}T_{00}^*\} \approx r_{00}^8$. Classes D and E are composed of SDMEs in which the main terms contain a product of $T_{10}$ (single helicity flip, $\gamma_L^* \to V_T$) and $T_{1-1}$ (double helicity flip, $\gamma_L^* \to V_T$), respectively, with $T_{11}^*$. Classes A and B contain seven SDMEs which should be different from zero for helicity non-flip (SCHC) transitions; as shown in Fig. 1, the measured SDMEs are significantly different from zero. The SDMEs of class A for the $\phi$ meson are $10 \text{-} 20\%$ larger than those for the $\rho^0$ meson. In terms of amplitudes this means that the modulus of the amplitude ratio $T_{11}/T_{00}$ is larger for the $\phi$ meson than for the $\rho^0$ meson. For SCHC to hold, the following three relations need to be fulfilled:

$$r_{1-1}^1 = -\text{Im}\{r_{1-1}^2\}, \quad \text{Re}\{r_{10}^5\} = -\text{Im}\{r_{10}^6\}, \quad \text{Im}\{r_{10}^8\} = \text{Im}\{r_{10}^7\}.$$ Figure 1 shows that they are approximately fulfilled both for $\phi$ and $\rho^0$ mesons. All SDMEs of the $\phi$ meson for class C fluctuate near zero, supporting the validity of SCHC. The SDMEs of class C show also pronounced differences between $\phi$ and $\rho^0$ mesons. For $\rho^0$ production, both $\text{Re}\{r_{04}^0\}$ and $r_{00}^5$ are found to be different from zero by several standard deviations for hydrogen and deuterium[2]. In $\phi$ production both SDMEs are found to be consistent with zero. From this observation we can draw simple conclusions: the SDME $r_{00}^5$ is proportional to the real part of the product of the amplitudes describing the transitions $\gamma_L^* \to V_L$ and $\gamma_L^* \to V_L$, with

$$r_{00}^5 \approx \text{Re}(T_{01}T_{00}^*) = |T_{01}| |T_{00}| \cos \delta_{01}, \quad (5)$$

where $\delta_{01}$ is the phase difference between the amplitudes $T_{01}$ and $T_{00}$. The real part of $T_{01}T_{00}^*$ can be close to zero if either $|T_{01}|$ is very small or $\delta_{01} \approx \pm \pi/2$. The first solution seems to be more likely as the SDME $r_{00}^8$, which is proportional to the imaginary part of the same product of amplitudes,

$$r_{00}^8 \approx \text{Im}(T_{01}T_{00}^*) = |T_{01}| |T_{00}| \sin \delta_{01}, \quad (6)$$

is compatible with zero within the experimental uncertainty. The amplitude $T_{01}$ provides information on the longitudinal quark motion in the vector meson. It was shown [3] that the amplitudes violating SCHC are zero if the two valence quarks have their longitudinal momenta equal one half of the meson momentum in the infinite momentum frame. The smaller value of $|T_{01}|$ for the $\phi$ meson compared to that for the $\rho^0$ meson means that the fractional longitudinal momenta of the two strange quarks in the $\phi$ meson are closer to each other and to half of the meson momentum than those of the $u$ and $d$ quarks in the $\rho^0$ meson. This means that the relative longitudinal quark motion is smaller in the $\phi$ meson than in the $\rho$ meson.

REFERENCES