Exclusive electroproduction of vector mesons in lepton nucleon scattering at the HERMES experiment

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Abstract

Exclusive electroproduction of vector mesons has been measured on hydrogen and deuterium targets at HERMES using the 27.6 GeV electron/positron beam of HERA. From this process, information can be obtained about generalized parton distributions (GPDs), which provide a unified description of the structure of hadrons embedding longitudinal-momentum distributions (ordinary PDFs) and transverse-position information (form factors). The study of the azimuthal distribution of the decay products via spin-density matrix elements provide constraints on helicity-amplitudes used to describe exclusive vector-meson production. Recent results from the HERMES experiment on the production of $\rho^0$, $\omega$ and $\phi$ vector mesons will be presented.

Keywords: Vector meson, Electroproduction, Spin Density Matrix Elements

1. Introduction and Definitions

Exclusive production of vector mesons was studied with the HERMES spectrometer at the DESY laboratory by scattering 27.6 GeV longitudinally polarized electrons or positrons off unpolarized hydrogen and deuterium as well as off transversely polarized hydrogen internal gas targets. The HERMES spectrometer was described in detail in Ref. [1]. In the forward spectrometer both the scattered lepton and the produced hadrons were detected within an angular acceptance $\pm 170$ mrad horizontally and $\pm (40-140)$ mrad vertically.

The kinematic region is defined by virtuality: $Q^2 = -(k - k')^2$, where $k$ and $k'$ are four-momenta of induced and scattered leptons; the squared invariant mass of photon nucleon system: $W^2 = (q + p)^2$, where $p$ is four-momentum of nucleon as well as $t' = t - t_0 = (q - v)^2$, where $v$ is four-momentum of vector meson and $-t_0$ represent the smallest kinematically allowed value of $-t$ at fixed $Q^2$ and fixed $v = E - E'$ - the difference of lepton energies. For HERMES it is: $1\text{GeV}^2 < Q^2 < 7\text{GeV}^2$, $3\text{GeV} < W < 10\text{GeV}$ and $-t' < 0.4\text{GeV}^2$.

A dual radiator RICH detector was used to identify hadrons in momentum range of 2 - 15 GeV. The exclusive region was defined by conditions on $\Delta E$: $-1\text{GeV} < \Delta E = M_{\pi}^2 - M_{p}^2 < 0.6\text{GeV}$, where $M_\pi$, $M_p$ are missing mass and mass of the target. The separation of the exclusive region and the contribution of semi-inclusive processes are presented in fig. 1.

Exclusive vector meson production in lepton-nucleon scattering provides a convenient tool for studying the vector meson production mechanism and also, in a model-dependent way, the structure of the nucleon. Exclusive vector meson electroproduction $e N \rightarrow e VN$ can be viewed as a sequence of the following processes: emission of a virtual photon by the incoming lepton; fluctuation of the virtual photon into a quark-antiquark pair; scattering of this $q \bar{q}$-pair off the nucleon; and formation of the final state [4]. Regge phenomenology and perturbative QCD (pQCD) provide complementary approaches to describe the rescattering of the $q \bar{q}$ pair off the nucleon. The interaction of the $q \bar{q}$ pair with the nucleon depends on the transverse separation between the $q$ and $\bar{q}$. A pair with large transverse sep-
Figure 1: Definition of the exclusive and semi-inclusive regions for φ-meson production. The vertical line separates the exclusive region (to left of line) from semi-inclusive one. The ∆E distribution for semi-inclusive processes is given by PYTHIA simulation [24].

The interaction is thought to interact primarily softly, which is described in Regge phenomenology [5] by the exchange of a pomeron or a secondary reggeon. The interaction of a q̅q pair with large transverse momentum and small transverse separation is calculable in pQCD. In lowest order of the strong coupling constant $\alpha_s$, this hard interaction is mediated by the exchange of a gluon-gluon or quark-antiquark system. At leading twist, exclusive production of mesons by longitudinally polarized virtual photons can be described within the "handbag" factorization scheme [6]. In the context of QCD, the non-perturbative features of the structure of the nucleon are described in terms of generalized parton distributions (GPDs): $H, \tilde{H}, E$ and $\tilde{E}$ [6, 8, 9].

For an unpolarized target and longitudinally polarized beam, the 3-dimensional angular distribution of production and decay of vector meson can be described in terms of Spin Density Matrix Elements (SDMEs). The relevant angles are defined in fig. 2. The $\cos(\theta)$ and azimuthal angle $\phi$ are determined in the vector meson rest frame. The angle between the scattering and production planes is defined in $(\gamma^*N)$ frame. The determination of 23 SDMEs will be presented and discussed. A wealth of information is contained in the spin density matrix elements (SDMEs), which are the observables describing how the spin components of the virtual photon are transferred to those of the vector meson created [2, 3]. Scattering longitudinally polarized electrons or positrons off unpolarized hydrogen and deuterium targets can be used to extract 23 SDMEs for the vector meson. The results are presented in the Schilling-Wolf [2] representation of SDMEs. Using the measured SDME values it is possible to test the hypothesis of $s$-channel helicity conservation (SCHC), which requires the produced vector meson to have the same helicity as the incident virtual photon. Non-conservation of $s$-channel helicity in $\rho^0$-meson production at the HERA collider was first observed by the H1 [10] and ZEUS [11] collaborations.

Additionally, the measured values of SDMEs can be used to study the hierarchy of natural-parity-exchange (NPE) amplitudes and also to explore the possible existence of contributions of unnatural-parity-exchange (UPE) amplitudes. NPE indicates that the interaction is mediated by a particle of ‘natural’ parity ($J^P = 0^+, 1^+, 2^+, \ldots$), while UPE amplitudes describe the exchange of a particle of ‘unnatural’ parity ($J^P = 0^-, 1^-, \ldots$).

The ratio of helicity amplitudes analysis is an important tool used for investigation of spin transitions $\gamma^* \rightarrow V$ [17]. The same angular distributions as those used to determine SDMEs were employed. The ratios of transition amplitudes to the main amplitude $T_{00}$ (index 0 (1) describe longitudinal (transverse) photon or vector meson), for which the factorization theorem has been proven. Amplitude ratios studies are useful tool for determination not only for transition of $\gamma^*_L \rightarrow V_L$ but also for ratios of small amplitudes describing spin-flip and UPE amplitudes as function of $Q^2$ or $t$.

There are also theoretical predictions for amplitudes ratios [3, 12, 19, 20] at large $Q^2$ and small $x_B$:

$$t_{11} = \frac{T_{11}}{T_{00}} \propto \frac{M_V}{Q} \cdot \frac{t_{01}}{T_{01}} \propto \frac{\sqrt{-t'}}{Q},$$
Here the SDME for $\rho^0$ is small. The ratio of helicity amplitudes will be discussed for $\rho^0$. The single-spin asymmetry on transversely polarized protons for $\phi$ and $\phi'$. The kinematic region for $\phi$ was limited to: $-1.0\text{GeV} < \Delta E < 0.8\text{GeV}; 1.0\text{GeV}^2 < Q^2 < 7.0\text{GeV}^2; 0.2\text{GeV}^2 < -t' < \text{GeV}^2$. In the case of $\rho^0$ and $\phi$ the same conditions were used: $-1.0\text{GeV} < \Delta E < 0.6\text{GeV}; 1.0\text{GeV}^2 < Q^2 < 7.0\text{GeV}^2; 0.0\text{GeV}^2 < -t' < \text{GeV}^2$.

2. Spin Density-Matrix Elements

In the Schilling–Wolf representation [2], for an unpolarized target the SDMEs $r^\alpha_{\lambda V, k'}$ are given by

$$r^\alpha_{\lambda V, k'} = \frac{1}{2N} \sum_{\lambda',k} F_{\lambda V, \lambda' k} \sum_{\alpha} F^\alpha_{\lambda', k} \frac{Q^2}{M^2} \left( C_1 \frac{t' M_V}{Q^2 + M^2} + C_2 \right),$$

(1)

where $N = N_T + \epsilon N_L$ is the normalization factor with $N_T$ and $N_L$ being proportional to the differential cross sections $d\sigma_T/dt$ and $d\sigma_L/dt$ for the transverse and longitudinal photons, and $\epsilon$ being the ratio of longitudinal to transverse virtual photon fluxes; $F_{\lambda V, \lambda' k}$ is the helicity amplitude of the subprocess $\gamma^*(\lambda')N(\lambda) \rightarrow V(\lambda')N'(\lambda_N)$, where $\gamma^*$ and $V$ are virtual photon and the produced vector meson and $N$ and $N'$ are the initial and final nucleons. Symbols in parentheses denote particle helicities. The nine Hermitian matrices $\Sigma^\alpha_{\lambda V, k}$ are defined in Ref. [2]. The upper index $\alpha = 0$ corresponds to an unpolarized transverse photon, $\alpha = 1, 2$ to the two directions of linear polarization, $\alpha = 3$ to circular polarization, $\alpha = 4$ to a longitudinal photon, and $\alpha = 5, \ldots, 8$ are attributed to the interference of longitudinal and transverse photons. At fixed lepton energy, as is the case in the present experiment, the matrices $r^4_{\lambda V, \lambda' k}$ and $r^0_{\lambda V, \lambda' k}$ cannot be extracted separately. Only their combination $r^4_{\lambda V, \lambda' k} = r^0_{\lambda V, \lambda' k} + \epsilon r^4_{\lambda V, \lambda' k}$ can be determined. With an unpolarized beam, the fifteen matrix elements $r^\alpha_{\lambda V, k}$ with photon polarization states $\alpha = 1, 2, 5, 6$ and $r^4_{\lambda V, \lambda' k}$ can be determined. They are referred to as ‘unpolarized’ SDMEs. The determination of the eight additional matrix elements $r^\alpha_{\lambda V, k}$ with $\alpha = 3, 7, 8$ is possible with the longitudinally polarized electron or positron beam at HERMES. These eight matrix elements are referred to as ‘polarized’ SDMEs. Each amplitude $F_{\lambda V, \lambda' k}$ may be decomposed into the sum of a NPE amplitude $T_{\lambda V, \lambda' k}$ and an UPE amplitude $U_{\lambda V, \lambda' k}$. For an unpolarized target, there is no interference between NPE and UPE amplitudes and SDMEs are defined as superposition of two products each involving only NPE or UPE amplitudes:

$$r^\alpha_{\lambda V, k} = \frac{1}{2N} \sum_{\lambda',k} \sum_{\alpha} \Sigma^\alpha_{\lambda V, k} \left[ T_{\lambda V, \lambda' k} T^\alpha_{\lambda', k} \right] + U_{\lambda V, \lambda' k} U^\alpha_{\lambda', k},$$

(3)

For an unpolarized target, contributions to SDMEs from NPE amplitudes with nucleon helicity flip are predicted to be suppressed by a factor of $(-t'/4M^2)$ [18]. Here, $t' = t - t_0$, where $t$ is the squared four-momentum transfer from the virtual photon to the vector meson, and...
Figure 4: The 23 SDMEs extracted for ω-meson production on proton (empty red squares) and deuteron (empty red circles) compared to ρ0 production on the proton (full red squares) and deuterium (full blue circles) [16] are presented. The inner error bars represent the statistical uncertainties, while the outer ones, where visibly larger, indicate the statistical and systematic uncertainties added in quadrature. The unshaded (shaded) areas indicate unpolarized (polarized) SDMEs. (in this figure second element is r1 not r11.) For easier interpretation the set of SDMEs was divided into five classes (see text). The dashed vertical line at zero indicates that zero value for elements from classes C, D and E is the case of s-channel helicity conservation.
$-t_0$ is the smallest kinematically allowed value of $-t$ at fixed $Q^2$ and fixed invariant mass $W$ of the $\gamma^*N$ system. Only the NPE amplitudes without nucleon helicity flip ($\lambda_N^* = \lambda_N$) will be discussed here and the abbreviated notation $T_{\lambda}A \equiv T_{\lambda}A_{\lambda \lambda}$ will be used.

In the HERMES kinematic region, the hierarchy of NPE helicity amplitudes

\[ |T_{00}| \sim |T_{11}| \gg |T_{01}| \sim |T_{1-1}| \]

(4)

was observed for $\rho^0$ production [16]. For high values of $W$, a similar hierarchy was originally proposed to exist for vector mesons [19]. It was experimentally confirmed, by ZEUS [11] for $\rho^0$ and H1 [10, 13] for $\phi$ productions, while UPE contributions were not observed as expected. At intermediate values of $W$ and $Q^2$, an UPE contribution was observed for $\rho^0$ production by HERMES [16] and interpreted as an indication of quark-antiquark exchange.

The SDMEs - $A_\lambda$ are parameters in the 3-dimensional vector meson production and decay angular distribution $W(\lambda, \cos \theta, \varphi, \Phi)$ which is defined above (see Fig. 2) [22]. The angular distribution $W(\lambda, \cos \theta, \varphi, \Phi)$ can be decomposed into terms describing unpolarized and a longitudinally polarized beam the angular distribution:

\[ W(\lambda, \cos \theta, \varphi, \Phi) = W^{UU}(\lambda^{UU}, \cos \theta, \varphi, \Phi) + P_B W^{LU}(\lambda^{LU}, \cos \theta, \varphi, \Phi), \]

(5)

where $W^{UU}$ corresponds to the case when both beam and target are unpolarized and $W^{LU}$ arises in the case of longitudinally polarized beam with polarization $P_B$. In fig. 4 the extracted SDMEs for $\omega$-meson on proton (empty red squares) and deuteron (empty red circles) and compared to $\rho^0$ production on proton (full red squares and full blue on deuteron) [16] are presented. The unshaded (shaded) areas indicate unpolarized - $A^{UU}$ (polarized - $A^{LU}$). For easier interpretation the set of SDMEs was divided into five classes. The dashed vertical line at zero indicates the case of s-channel helicity conservation (SCHC) for classes C, D, E. The classes A and B include contributions from amplitudes corresponding to transitions: $\gamma^*_L \rightarrow V_L$ and $\gamma^*_T \rightarrow V_T$ . The elements from class A contains the squared of the amplitudes: $|T_{00}|^2$, $|T_{11}|^2$, $|U_{11}|^2$, . . . , while class B contains the interference terms of amplitudes like : $(T_{11} - T_{T-1})T_{00}$. Classes from C to E contain dominant contributions of amplitudes with spin-flip transitions $\lambda_T \neq \lambda_T$. Class C contains SDMEs with dominant terms that are products of s-channel helicity non-conserving amplitude $T_{01}$ corresponding to the $\gamma^*_T \rightarrow V_L$ transition and $T_{00}$, $T_{11}$ . . .

Classes D and E are composed SDMEs in which the main terms contain a product of $T_{10}$ (single helicity-flip, $\gamma^*_L \rightarrow V_T$) and $T_{1-1}$ (double helicity-flip, $\gamma^*_T \rightarrow V_{-T}$), respectively, with $T_{11}$. As can be seen in fig. 4 the SDMEs for $\omega$ meson: $r_{10}^2 \sim |T_{10}|^2 + |T_{1-1}|^2$ - $(|U_{11}|^2 + |U_{-11}|^2)^2$ and $\text{Im}[r_{11}^2] \sim -|T_{11}|^2 + |T_{1-1}|^2 + |U_{11}|^2 - |U_{-11}|^2$ have the opposite signs than the same elements for $\rho^0$. It

\[ R = \sigma_L / \sigma_T \]

is fullfil when $(|U_{11}|^2 + |U_{-11}|^2) > (|T_{11}|^2 + |T_{1-1}|^2)$

\[ 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2 \]

\[ 0.05, 0.1, 0.15, 0.2 \]

\[ 0, 0.05, 0.1, 0.15, 0.2 \]

\[ 0, 0.2, 0.4 \]

\[ 0, 0.2, 0.4 \]

\[ 1, 2 \]

\[ 1, 2 \]
and agree with hierarchy inequality $4 |U_{11}|^2 > |T_{11}|^2$. The quantity $U_{11}$ define contribution of UPE processes: $U_{11}=1-r_0^4+2r_0^4-2r_1^4-2r_{11}^4$. It may be expressed by UPE amplitudes: $U_{11}=\Sigma[4\epsilon[U_{00}^2+2|U_{11}+U_{1-1}|^2]/N$, where $N$ is normalization constant. In fig. 5 the signature $U_{11}$ is presented as a function of $Q^2$ and $t'$. The values of $U_{11}$ are higher for $\omega$ vector meson than for $\rho^0$. Also, from fig. 6 it can be seen the ratio $R=\sigma_{U_{11}}/\sigma_{T}$ is small compare to that for $\rho^0$. The element $r_0$ sensitive to transitions $\gamma_T \rightarrow V_L$ distinctive from zero for $\rho^0$ is not observed in the case of $\omega$.

3. Ratios of helicity amplitudes for $\rho^0$

In the previous section the method of determination of SDMEs $\omega$ vector meson in Schilling-Wolf representation was presented. The angular distribution given by eq. 5 were used to extract the ratios of helicity amplitudes. As it was shown in [17] the free parameters of angular distributions can be defined as ratios of amplitudes in eq. 1. The number of free parameters will be reduced to nine, containing real and imaginary parts of amplitudes (see eq. 1) and amplitude $u_{11}$. Reduced number of free parameters gives a possibility to determine the dependences on $Q^2$ or $t'$. In previous section the UPE for $\omega$ was discussed. Therefore, the ratio of amplitude and 8. The fitting result gives a constant value for both $Q^2$ and $t'$ dependence.

The unnatural-parity-exchange amplitude $U_{11}$ describes the transition from transversely polarized photon to transversely polarized $\rho^0$ meson ($\gamma_T \rightarrow \rho^0 T$). At large $W$ and $Q^2$, this transition should be suppressed by factor of $M_V/Q$ compared to dominant amplitude $T_{00}$. The UPE contributions to this amplitude may be sizable at intermediate energies typical for HERMES. It was found that the predicted asymptotic dependence on $Q^2$ of real part of amplitudes $t_{11}$ agrees with experiment for proton and deuteron, but the imaginary part rise with $Q^2$. For $t_{00}$ the others functions describing $Q^2$ and $t'$ dependences have been found: $\text{Im}(t_{00})=\sqrt{-t'}(f_1+f_2Q^2)$, $f_1=0.663 \pm 0.132 \text{ GeV}^{-1}$ and $f_2=-0.285 \pm 0.065 \text{ GeV}^{-3}$ as well as $\text{Re}(t_{00})=c\sqrt{-t'}$ with $c=0.394\pm 0.024 \text{ GeV}^{-1}$.

![Figure 7](image7.png)

Figure 7: Dependence of the ratio $|U_{11}/T_{00}|$ on $Q^2$ for proton and deuteron. The inner errors bars show the statistical uncertainty and outer ones shows the statistical and systematic added in quadrature.

$|U_{11}/T_{00}|$ as function of $Q^2$ is presented first in figs. 7 and 8. The fitting result gives a constant value for both $Q^2$ and $t'$ dependence.

4. Single-Spin Asymmetry in Exclusive Electroproduction of $\phi$ and $\rho^0$ Meson on Transversely Polarized Protons

The data with transversely polarized hydrogen target were collected during the period 2002 -2005. The spin direction was reversed every 1-3 minutes. The average magnitudes of target polarization was $\langle |P_T| \rangle = 0.724 \pm 0.059$. 

![Figure 8](image8.png)

Figure 8: Dependence of the ratio $|U_{11}/T_{00}|$ on $t'$ for proton and deuteron. The inner errors bars show the statistical uncertainty and outer ones shows the statistical and systematic added in quadrature.
The master formula (34) of ref. [30] expressing the differential cross section as sum of functions: \( \Sigma A_{ij} \sin(\phi \pm j\phi_S) \) for \( PT = 0 \) was used.

\[
\left( \frac{\cos \theta}{1 - \sin^2 \theta \sin^2 \phi_S} \right)^{-1} \left( \frac{\alpha_{em}}{8\pi^3} \frac{y^2}{1 - e} \frac{1}{x_B} \frac{1}{Q^2} \right)^{-1} \times \frac{d\sigma}{dxdt}\bigg|_{\text{full range}} =
\]

- terms independent of \( PT \)

\[
- \frac{P_T}{\sqrt{1 - \sin^2 \theta \sin^2 \phi_S}} \sin \phi_S \cos \theta \sqrt{1 + e} \text{Im} \sigma^+_{\epsilon_0^-} + \sin \phi - \phi_S \right) \left( \cos \theta \text{Re} \sigma^+_{\epsilon_0^-} + e \text{Re} \sigma^+_{\epsilon_0^-} \right)
+ \frac{1}{2} \sin \theta \sqrt{1 + e} \text{Im} \sigma^+_{\epsilon_0^-} + \frac{1}{2} \sin \theta
+ \sqrt{1 + e} \text{Im} \sigma^+_{\epsilon_0^-} - \sigma^-_{\epsilon_0^-})
+ \sin(2\phi - \phi_S) \left( \cos \theta \sqrt{1 + e} \text{Im} \sigma^-_{\epsilon_0^-} + \frac{1}{2} \sin \theta \text{Re} \sigma^+_{\epsilon_0^-} \right)
+ \frac{1}{2} \sin \theta \text{Re} \sigma^+_{\epsilon_0^-}
+ \sin(2\phi + \phi_S) \frac{1}{2} \sin \theta \text{Re} \sigma^+_{\epsilon_0^-}
+ \sin(3\phi - \phi_S) \cos \theta \frac{1}{2} \text{Im} \sigma^+_{\epsilon_0^-}\right) \]

(6)

Upper indexes describe proton polarization and lower photon polarization (details see ref. [30]). In the HERMES kinematics the angle \( \theta \) is small reducing the second term. Only one amplitude of \( \sin(\phi - \phi_S) \) has physical interpretation and was calculated assuming shape and parameters function \( E \) of GPD.

The \( A_{UT} \) for \( \rho^0 \) was obtained as a combination of SDMEs determined from data with transversely polarized target [3]. Likewise in the case of SDMEs with polarized lepton beam the angular distributions, with added \( \phi_S \), were used (see eq. 7). \( \Phi \) is the angle between the scattering and production planes with opposite sign use in Shilling-Wolf representation, \( -\Phi = \phi \) agree with [3] \( \phi \). The 15 elements of SDMEs for unpolarized data set (set of data with helicity balanced beam) was determined first. With fixed 15 unpolarized elements, the 30 elements related with transversely polarized target were determined. The \( A_{UT} \) from eqs. 8 for longitudinally and transversely polarized photon were calculated. The results are presented in fig. 10.

\[
W(\lambda, \cos \theta, \varphi, -\Phi, \phi_S) = W^{LU}(\lambda^{LU}, \cos \theta, \varphi, -\Phi)
+ P_T W^{UT}(\lambda^{UT}, -\Phi, \phi_S), \quad (7)
\]

\[
A_{UT}^{L, \sin(\phi - \phi_S)} = \frac{\text{Im}(\alpha_{00}^+ + \epsilon_{00}^0)}{\alpha_{00}^+ + \epsilon_{00}^0},
A_{UT}^{T, \sin(\phi - \phi_S)} = \frac{\text{Im}(\alpha_{00}^+ + \epsilon_{00}^0 + 2\epsilon_{00}^+)}{1 - (u_{00}^0 + \epsilon_{00}^0)}. \quad (8)
\]

The experimental value of \( A_{UT}^{L, \sin(\phi - \phi_S)} = -0.035 \pm \)
A few groups have performed GPD-based calculations of the transverse asymmetry for exclusive $\rho^0$ production. In refs. [7, 31] the quarks GPD $E^q$ is parametrized in terms of the values of $J^u$ taking $J^d = 0$. In ref. [31] contribution of gluon was included. The calculated value $A_{LL}^{\sin(\phi - \phi_S)} U_T$ are in range 0.15-0.0 for $J^u = 0$ to 0.4. In ref. [25, 32] the GPD are modeled using the data for nucleon form factor, sum rules and positivity constraints. The results of both calculations are similar. Values of $J^u$ and $J^d$ of approximately 0.22 and 0.0. The calculated values of asymmetry are very small: from -0.03 to 0.02. The contributions of $u$ and $d$ quarks for $\rho^0$ asymmetry are different. Different contributions from sea-quarks, valence quarks and gluons for particular function are expected. Models for GPDs, particularly for function $E$ and prediction of $A_{UT}$ for production of different vector models has been done in ref. [25]. In our case sea-quarks and gluons in GPDs are important. For $H$ function of GPD small contribution are introduced from sea-quarks compared to valence quarks. In the case of $\phi$ meson the predicted asymmetry should be 0.0. The gluon and sea-quark moments have to cancel almost completely is the conclusion from ref. [25]. Our results for $\phi$ meson in spite of the errors confirms this result.

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