Extraction of helicity amplitude ratios from exclusive $\rho^0$-meson electroproduction on transversely polarized protons

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Abstract. Exclusive $\rho^0$-meson electroproduction is studied by the HERMES experiment, using the 27.6 GeV longitudinally polarized electron/positron beam of HERA and a transversely polarized hydrogen target, in the kinematic region $1.0 \text{ GeV}^2 < Q^2 < 7.0 \text{ GeV}^2$, $3.0 \text{ GeV} < W < 6.3 \text{ GeV}$, and $-t' < 0.4 \text{ GeV}^2$. Using an unbinned maximum-likelihood method, 25 parameters are extracted. They determine the real and imaginary parts of the ratios of certain helicity amplitudes (describing $\rho^0$-meson production by a virtual photon) and the dominant amplitude $F_{0^+0^+}$ without the nucleon-helicity flip. The latter amplitude describes the production of a longitudinal $\rho^0$ meson by a longitudinal virtual photon. The transverse target polarization allows for the first time the extraction of ratios of a number of nucleon-helicity-flip amplitudes to $F_{0^+0^+}$. The ratios of nucleon-helicity-non-flip amplitudes to $F_{0^+0^+}$ are found to be in good agreement with those from the previous HERMES analysis. A comparison of the extracted amplitude ratios with the Goloskokov-Kroll model shows the necessity to add pion exchange amplitudes with positive $\pi\rho$ form factor to the amplitudes based on generalized parton distributions to improve the HERMES data description.

1. Introduction

Exclusive electroproduction of vector mesons ($V$) on nucleons ($N$) gives information both on the reaction mechanisms and the nucleon structure [1]. Electroproduction at high energies can be considered to consist of three subprocesses: i) the incident lepton emits a virtual photon $\gamma^*$, which dissociates into a quark-antiquark pair; ii) this $q\bar{q}$ pair interacts strongly with the nucleon; iii) the observed vector meson is formed from the scattered $q\bar{q}$ pair. In Regge phenomenology, the interaction of the $q\bar{q}$ pair with the nucleon proceeds through the exchange of a pomeron or/and exchange of secondary reggeons. If the quantum numbers of the particle lying on the Regge trajectory are $J^P = 0^+, 1^-$, etc. (pomeron, $\rho$, $f_2$, ...), the process is denoted Natural Parity Exchange (NPE). Alternatively, the case of $J^P = 0^-, 1^+$, etc. ($\pi$, $a_1$, ...) corresponds to Unnatural Parity Exchange (UPE). In the framework of perturbative quantum chromodynamics valid at large photon virtuality $Q^2$ and high photon-nucleon center-of-mass (CM) energy $W$, the nucleon structure can also be studied through hard exclusive meson production, as the process amplitude contains Generalized Parton Distributions (GPDs) (see review [1]). However, the factorization property that permits to extract GPDs is rigorously proved only for the amplitudes $F_{0^+0^+}$ of longitudinal vector meson production by longitudinal virtual photons [2]. In the Goloskokov-Kroll (GK) model (see [3] and references therein), the validity of factorization is...
assumed for some other amplitudes in addition to $F_{\rho}^{0 \pm 0 \pm \pm}$ and this assumption is justified with a good description of the existing data. The presented HERMES data can be well described in the GK model if the pion exchange is taken into account (see [4] and references therein). This means that the GPD-based approach should be modified at intermediate energies and $Q^2$.

All observables in vector-meson electroproduction can be expressed in terms of the helicity amplitudes in the CM system describing $\rho^0$-meson production by a virtual photon. In particular, Spin-Density-Matrix Elements (SDMEs) are functions of the helicity amplitude ratios (HARs). Therefore HARs can be extracted from the data as was shown in Ref. [5]. For the first time, HARs with nucleon-helicity flip are obtained in the present analysis since the data on $\rho^0$-meson production by the longitudinally polarized electrons/positrons on a transversely polarized hydrogen target are accumulated. The most recent HERMES results on the $\rho^0$-meson production presented at this workshop are published in Ref. [6] where more details can be found.

2. Spin-density-matrix elements and helicity amplitudes

The SDMEs are the Fourier coefficients in the angular distribution of the final-state particles. In the present paper, the formalism proposed in Ref. [7] for SDMEs is used. Any SDME can be expressed as a ratio of two sums of bilinear products of the helicity amplitudes $F_{\lambda_V,\lambda',\lambda,\lambda_N}$ and their complex conjugate quantities $F^*_{\lambda_V,\lambda',\lambda,\lambda_N}$. Here, $F_{\lambda_V,\lambda',\lambda,\lambda_N}$ is the helicity amplitude of the process $\gamma^*(\lambda_\gamma) + N(\lambda_N) \to V(\lambda_V) + N(\lambda_N')$, where the particle helicities are given in parentheses. The helicity amplitudes depend on $W$, $Q^2$, and $t' = t - t_{\text{min}}$, where $t$ is the Mandelstam variable and $-t_{\text{min}}$ represents the smallest kinematically allowed value of $-t$ at fixed $W$ and $Q^2$.

Any helicity amplitude can be decomposed into a sum of a NPE ($T_{\lambda_V,\lambda',\lambda,\lambda_N}$) and an UPE ($U_{\lambda_V,\lambda',\lambda,\lambda_N}$) amplitude: $F_{\lambda_V,\lambda',\lambda,\lambda_N} = T_{\lambda_V,\lambda',\lambda,\lambda_N} + U_{\lambda_V,\lambda',\lambda,\lambda_N}$, for details see Refs. [7, 8]. The amplitudes obey the symmetry relations that hold because of parity conservation

$$T_{\lambda_V,\lambda',\lambda,\lambda_N} = (-1)^{-\lambda_N + \lambda_N'} T_{-\lambda_V,\lambda',-\lambda,\lambda_N} = (-1)^{-\lambda_N + \lambda_N'} T_{\lambda_V,-\lambda',\lambda,-\lambda_N},$$

$$U_{\lambda_V,\lambda',\lambda,\lambda_N} = -(1)^{-\lambda_N + \lambda_N'} U_{-\lambda_V,\lambda',-\lambda,\lambda_N} = (1)^{-\lambda_N + \lambda_N'} U_{\lambda_V,-\lambda',\lambda,-\lambda_N}.$$  \hspace{1cm} (1)

These symmetry properties of the helicity amplitudes permit to introduce the notations:

$$T^{(1)}_{\lambda_V,\lambda} \equiv T_{\lambda_V,\frac{1}{2},\lambda,\frac{1}{2}}, \quad U^{(1)}_{\lambda_V,\lambda} \equiv U_{\lambda_V,\frac{1}{2},\lambda,\frac{1}{2}}, \quad T^{(2)}_{\lambda_V,\lambda} \equiv T_{\lambda_V,\frac{1}{2},\lambda,-\frac{1}{2}}, \quad U^{(2)}_{\lambda_V,\lambda} \equiv U_{\lambda_V,\frac{1}{2},\lambda,-\frac{1}{2}}.$$  \hspace{1cm} (2)

All other amplitudes can be obtained from Eqs. (1–2). The HARs extracted in the present analysis are defined for $n = 1, 2$ as $t^{(n)}_{\lambda_V,\lambda} = T^{(n)}_{\lambda_V,\lambda}/T^{(1)}_{\lambda_V,\lambda}$, $u^{(n)}_{\lambda_V,\lambda} = U^{(n)}_{\lambda_V,\lambda}/T^{(1)}_{\lambda_V,\lambda}$.

3. Experiment and data analysis

3.1. Experiment

The data were accumulated with the HERMES spectrometer using the 27.6 GeV longitudinally polarized electron or positron beam of HERA and a gaseous hydrogen target. The HERMES setup included a forward spectrometer [9], in which the scattered lepton and the produced hadrons were detected within an angular acceptance of $\pm 170$ mrad horizontally and $\pm (40 - 140)$ mrad vertically. The tracking system had a momentum resolution of about 1.5% and an angular resolution of about 1 mrad. Lepton identification was accomplished using a transition-radiation detector, a preshower scintillator counter, and an electromagnetic calorimeter. The particle-identification system included also a dual-radiator ring-imaging Cherenkov detector [10] to identify hadrons. Combining the responses of the detectors in a likelihood method leads to an average lepton-identification efficiency of 98%, with a hadron contamination of less than 1%.

The helicity of the beam was typically reversed every two months. The beam polarization was continuously measured by two Compton polarimeters [11, 12]. The average value of the beam
polarization for the events used in the analysis is ±0.30 with a 0.03 uncertainty. A small part of the data was collected with an unpolarized target and the main part with a transversely polarized target [13], for which the polarization direction was reversed every 60 s to 180 s. The measured mean value of the target polarization is 

3.2. Data analysis
The \( \rho_0 \) mesons are produced and decay in the following exclusive reactions: \( e + p \rightarrow e + p + \rho_0 \), and \( \rho_0 \rightarrow \pi^+ + \pi^- \). The event sample used in this analysis is practically the same as that used in Ref. [14]. The most important improvement is the application of a new tracking algorithm, which is based on a Kalman filter [15]. For the present analysis, the data are required to fulfil the following criteria: i) The scattered lepton has to have an energy larger than 3.5 GeV in order to not introduce e

v) The two-pion invariant mass is required to obey \( 0.6 \text{ GeV} \leq m_{\pi^+\pi^-} \leq 1.0 \text{ GeV} \).

The recoiling proton was not detected, but instead reconstructed through the missing energy. Taking into account the spectrometer resolution, the missing energy \( \Delta E \) has to lie in the interval -1.0 GeV \( \leq \Delta E \leq 0.8 \text{ GeV} \). Here, \( \Delta E = M_\pi^2 - M_p^2 \), with \( M_p \) being the proton mass and \( M_\pi^2 = (p + q - p_{\pi^+} - p_{\pi^-})^2 \) the missing mass squared, where \( p, q, p_{\pi^+}, \) and \( p_{\pi^-} \) are the four-momenta of target nucleon, virtual photon, and each of the two pions respectively.

The distribution of missing energy \( \Delta E \) (shown in Ref. [14], exhibits a clearly visible exclusive peak.

The kinematic constraints \( Q^2 > 1.0 \text{ GeV}^2 \), \( 6.3 \text{ GeV} < W < 3 \text{ GeV} \), \(-t' < 0.4 \text{ GeV}^2 \) are applied. After application of all these requirements, the data sample contains 8741 events.

These data are referred to in the following as data in the “entire kinematic region”. The applied constraints do not fully suppress the background. The exclusive sample contains contributions from double-diffractive processes, which should be negligible in the low \( \Delta E \) region, from non-resonant \( \pi^+\pi^- \) pair production, which is of the order of 1 – 2% [14], from \( \Delta \) isobar excitation, which in the HERMES acceptance is less than 7% [16] and from semi-inclusive deep-inelastic scattering (SIDIS) events. The presented results are not corrected for the former three processes, while a correction is applied for SIDIS background.

The shaded histogram of missing energy is corrected for polarized target [13], for which the polarization direction was reversed every 60 s to 180 s. The missing energy for the events used in the analysis is less than 7% [16] and from semi-inclusive deep-inelastic scattering (SIDIS) events. The presented results are not corrected for the former three processes, while a correction is applied for SIDIS background. The function \( \rho \) is fixed according to the results of Refs. [17, 18], its modulus is fit so that Re[\( u_{11}^{(1)} \)] and Im[\( u_{11}^{(1)} \)] represent the results for

4. Results
Using an unbinned maximum-likelihood method the HARs are obtained from the 25-parameter fit in the entire kinematic region \((W) = 4.73 \text{ GeV}, \langle Q^2 \rangle = 1.93 \text{ GeV}^2, \langle -t' \rangle = 0.132 \text{ GeV}^2 \).

The result is shown in Fig. 1 with red points. While the phase of \( u_{11}^{(1)} \) is fixed according to the results of Refs. [17, 18], its modulus is fit so that Re[\( u_{11}^{(1)} \)] and Im[\( u_{11}^{(1)} \)] represent the results for
earlier in Ref. [5] for the unpolarized proton. All other ratios of nucleon-helicity-flip amplitudes area corresponds to ratios of amplitudes without the nucleon-helicity flip obtained here and one free parameter. The systematic uncertainty of Re[$u_{11}^{(1)}$] is much larger than the statistical one due to the large total experimental uncertainty of the $u_{11}^{(1)}$ phase. The value of Im[$t_{11}^{(1)}$] represents the result of Ref. [5]; its error bar shows the total uncertainty. The shadowed area corresponds to ratios of amplitudes without the nucleon-helicity flip obtained here and earlier in Ref. [5] for the unpolarized proton. All other ratios of nucleon-helicity-flip amplitudes to $T_{40}^{(1)}$ are calculated for the first time. As can be seen from Fig. 1 all HARs except $t_{11}^{(1)}$, $t_{01}^{(1)}$, and $u_{11}^{(1)}$ are compatible with zero within two standard deviations of their total uncertainty. The HAR $t_{01}^{(1)}$ is responsible for the violation of the s-channel helicity-conservation approximation, while $u_{11}^{(1)}$ shows the role of UPE in $\rho^0$-meson electroproduction.

The results of the theoretical predictions in the Goloskokov-Kroll model [3, 19] are depicted with blue triangles and squares in Fig. 1. In order to describe the presented HERMES data the one pion exchange (OPE) diagrams have to be included in addition to the handbag Feynman graphs containing the GPDs. Note that even the HARs compatible with zero are informative and can be used to fix the sign of the $\pi\rho$ form factor $g_{\pi\rho}(Q^2)$ in the OPE amplitude which is proportional to $g_{\pi\rho}(Q^2)/(t - m_\rho^2)$, where $m_\rho$ denotes the $\rho^0$ mass. As seen from Fig. 1 the positive sign of the $\pi\rho$ form factor $g_{\pi\rho}(Q^2)$ (triangles) corresponds to the extracted HARs $u_{11}^{(2)}$ and $u_{10}^{(2)}$, that are sensitive to the sign, much better ($\chi^2/ndf = 1.8/4$) than the negative sign.

Figure 1. Comparison of amplitude ratios determined in Ref. [6] to those calculated in the GK model. The red, filled circles correspond to the extracted amplitude ratios and the blue, open triangles (squares) represent the result of the GK model calculation using the positive (negative) sign of the $\pi\rho$ transition form factor. The inner (outer) error bars of red points represent the statistical (total) uncertainty. The amplitude ratios that are set to zero in the GK model are not shown. The amplitude ratios are ordered according to the classes proposed in Ref. [8, 14].
Figure 2. Comparison of the Diehl SDMEs [7] \( n_{\gamma V}^{\lambda V \lambda V} \) calculated from the helicity-amplitude ratios (red circles) and the SDMEs (blue squares) directly extracted in Ref. [14] in the entire kinematic region. For the first case, a 25-parameter fit is used. The points in the shaded area show SDMEs that can be obtained only if the beam is longitudinally polarized. The inner (outer) error bars represent the statistical (total) uncertainty.

(squares) for which \( \chi^2/ndf = 30.3/4 \).

The most serious disagreement between the extracted HAR \( \text{Im}[u^{(1)}_1] \) and that calculated in the GK model was already discussed in the previous HERMES work [5] and can be explained probably with the unsatisfactory description of the \( q\bar{q} \) scattering process from the proton in the GK model. As can be seen from Fig. 1 neither the phase nor the modulus of the amplitude ratio \( u^{(1)}_1 \) are described well in the GK model and this problem should be solved in the future.

A comparison of the SDMEs directly extracted from the HERMES data in Refs. [8] and [14] to those calculated with the HARs extracted in the present analysis is performed in Ref. [6] and good agreement is found. The correlation matrix for the 25 parameters is taken into account for the calculation of the statistical uncertainty of the SDMEs in the Diehl representation [7] \( u_{\lambda V, \lambda V}^{\lambda V \lambda V} \), \( n_{\lambda V, \lambda V}^{\lambda V \lambda V} \), and \( s_{\lambda V, \lambda V}^{\lambda V \lambda V} \) obtained from the HARs. The SDMEs \( u_{\lambda V, \lambda V}^{\lambda V \lambda V} \) and \( n_{\lambda V, \lambda V}^{\lambda V \lambda V} \), presented in Figs. 2 and 3, have linear contributions of the small HARs \( \xi_{\lambda V, \lambda V}^{(2)} \) and \( u_{\lambda V, \lambda V}^{(2)} \), respectively. These SDMEs can only be extracted from measurements with a transversely polarized target so that the nucleon-helicity-flip amplitude ratios \( \xi_{\lambda V, \lambda V}^{(2)} \) and \( u_{\lambda V, \lambda V}^{(2)} \) are extracted in Ref. [6] for the first time. The total uncertainty is the sum in quadrature of the statistical and the total systematic uncertainties. The SDMEs in Figs. 2 and 3 are reordered according to the SDME classes proposed in Refs. [8, 14]. Those SDMEs that can be extracted only from data taken with a longitudinally polarized lepton beam are shown in shaded areas. Figures 2 and 3 show that for some of the calculated SDMEs \( n_{\lambda V, \lambda V}^{\lambda V \lambda V} \) and \( s_{\lambda V, \lambda V}^{\lambda V \lambda V} \), no published results from Ref. [14] exist, because the beam polarization was not exploited in that analysis. While in Refs. [8] and [14] a total of 53 SDMEs could be extracted, the amplitude method presented here allows
Figure 3. Comparison of the Diehl SDMEs [7] $\lambda_V^\lambda_\gamma V'$ calculated from the helicity-amplitude ratios (red circles) and the SDMEs (blue squares) directly extracted in Ref. [14]. The meaning of the error bars and the further explanations are the same as for Fig. 2.

for the calculation of 71 SDMEs based on the extraction of 25 parameters. As seen from Figs. 2 and 3, there is reasonable agreement between SDMEs directly extracted in Refs. [8, 14] and those calculated from the HARs in Ref. [6]. The SDMEs $u_{\lambda^*_\gamma \lambda'_\gamma}^{\lambda_V}$ which can be obtained with unpolarized targets are not shown in the present paper. But a comparison of the calculated SDMEs $u_{\lambda^*_\gamma \lambda'_\gamma}^{\lambda_V}$ with those directly extracted from the HERMES data [8] is also performed in Ref. [6] (for the first time in Ref. [5]) and shows reasonable agreement.

References

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