Transverse-momentum distributions in electro-production and $e^+e^-$ annihilation

selected results from HERMES as well as BaBar, Belle and BESIII
what makes the visible universe

**Standard Model**

### 3 generations of fermions

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (MeV)</td>
<td>charge</td>
<td>spin</td>
</tr>
<tr>
<td>2.4</td>
<td>2/3</td>
<td>1/2</td>
</tr>
<tr>
<td>1.27</td>
<td>2/3</td>
<td>1/2</td>
</tr>
<tr>
<td>171.2</td>
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</tr>
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<td>u</td>
<td>c</td>
<td>t</td>
</tr>
<tr>
<td>up</td>
<td>charm</td>
<td>top</td>
</tr>
</tbody>
</table>

### Quarks

- d: 4.8 MeV, -1/3, 1/2
- s: 104 MeV, -1/3, 1/2
- b: 4.2 GeV, -1/3, 1/2

### Leptons

- e: 0.511 MeV, -1, 1/2
- μ: 105.7 MeV, -1, 1/2
- τ: 1.777 GeV, -1, 1/2

### Gauge Bosons (forces)

- Higgs: 125 GeV

+ Higgs
what makes the visible universe

standard model

- not Higgs but non-abelian quantum chromodynamics (QCD) responsible for mass in every-day life
- 40+ years success story of (p)QCD
- but non-perturbative part (hadron structure and formation) still a vast, partly unexplored field
4+ decades of QCD - a success story

\[ F_p^2 \cdot c \]

[A. Airapetian et al., JHEP 05 (2011) 126]

\[ \langle x \rangle \]

\[ c \]

\[ 0.008 \ 1.6^{40} \]
\[ 0.011 \ 1.6^{39} \]
\[ 0.015 \ 1.6^{38} \]
\[ 0.019 \ 1.6^{37} \]
\[ 0.025 \ 1.6^{36} \]
\[ 0.033 \ 1.6^{35} \]
\[ 0.040 \ 1.6^{34} \]
\[ 0.049 \ 1.6^{33} \]
\[ 0.060 \ 1.6^{32} \]
\[ 0.073 \ 1.6^{31} \]
\[ 0.089 \ 1.6^{30} \]
\[ 0.108 \ 1.6^{29} \]
\[ 0.134 \ 1.6^{28} \]
\[ 0.166 \ 1.6^{27} \]
\[ 0.211 \ 1.6^{26} \]
\[ 0.273 \ 1.6^{25} \]
\[ 0.366 \ 1.6^{24} \]
\[ 0.509 \ 1.6^{23} \]
\[ 0.679 \ 1.6^{22} \]

\( Q^2 [ \text{GeV}^2] \)
4+ decades of QCD - a success story?

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma \]
\[ + \Delta G \]
\[ + L_q + L_g \]

- quark spin \( \approx \frac{1}{2} \frac{1}{3} \)
- gluon spin \( \approx 0 / \text{small} \)
- orbital angular momentum \( \approx ? \)

- spin of the nucleon
4+ decades of QCD - a success story?

\[ A_N \propto \alpha_S \frac{m_q}{Q^2} \]

- spin of the nucleon
- transverse single-spin asymmetries in pp collision

\[ \sqrt{s} = 4.9 \text{ GeV} \]

\[ \text{ANL} \]

\[ 1976 \]

\[ \text{RHIC} \]

\[ 2008 \]

\[ \text{62.4 GeV} \]
4+ decades of QCD - a success story?

- spin of the nucleon
- transverse single-spin asymmetries in pp collision
- hadron-momentum preference in hadronization of pol. quarks
Deviation from Lam-Tung relation in NNLO

\[ \left( \frac{1}{\sigma} \right) \left( \frac{d\sigma}{d\Omega} \right) = \left[ \frac{3}{4\pi} \right] \left[ 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right] \]

\[ 1 - \lambda - 2\nu = 0 \]

- spin of the nucleon
- transverse single-spin asymmetries in pp collision
- hadron-momentum preference in hadronization of pol. quarks
- violation of Lam-Tung relation in Drell-Yan

4+ decades of QCD - a success story?
4+ decades of QCD - a success story?

- spin of the nucleon
- transverse single-spin asymmetries in pp collision
- hadron-momentum preference in hadronization of pol. quarks
- violation of Lam-Tung relation in Drell-Yan
- large hyperon polarization in unpolarized proton collisions
Generic pp data - x and p T dependence turns out to be negative for p T above 1 GeV/c. p T becomes flat (measured up to p T = 4 GeV/c).

DIS 2010, Florence, April 21, 2010

Deviation from Lam-Tung relation in NNLO:

- O(α_s^2) pQCD is (at least) an order of magnitude smaller and of opposite sign [Brandenburg, Nachtmann & Mirkes '93; Mirkes & Ohnemus '95]

With collinear parton densities, only higher order gluon emission can generate deviations from Lam-Tung.

Failure of collinear pQCD treatment:

- Measurement of A_N in pp-scattering for different center of mass energies:
  - 1976: 4.9 GeV
  - 2002: 6.6 GeV
  - 1991: 19.4 GeV
  - 2008: 62.4 GeV

- Only two models consistently describing the data:
  - TMDs (Transverse Momentum Dependent) distributions
  - high-twist correlations

- Interpretation not yet completely satisfactory

- All available models predict A_N goes to zero at high p T values.

- BUT: not yet DATA at such kinematic region

- All available data coming from pp scattering

MOTIVATION

Alejandro López Ruiz
Universiteit Gent
Florence/DIS 10

SSA in inclusive hadron production at HERMES

ANL
BNL
FNAL
RHIC
Generic data - $F$ and $p_T$ dependence turns out to be negative for $p_T$ above 1 GeV/c. $p_T$ becomes flat (measured up to $p_T^4$ GeV/c).

**MOTIVATION**

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SSA in inclusive hadron production at HERMES

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$A_N = N_R - N_L$  
$N_R + N_L$  

Only two models consistently describing the data:

* TMDs (Transverse Momentum Dependent) distributions  
* high-twist correlations

Interpretation not yet completely satisfactory

All available models predict $A_N$ goes to zero at high $p_T$ values.

**BUT**: not yet DATA at such kinematic region

All available data coming from $p p$ scattering.
transverse-momentum distributions

\[ \vec{k}_T \]

Transverse plane

Longitudinal momentum

\[ k^+ = x P^+ \]

Transverse momentum

partons

[courtesy of A. Bacchetta, Pavia]
transverse-momentum distributions

- go beyond collinear description of original quark-parton model
- explore correlations between spins and transverse momenta
- new insights into workings of QCD

- e.g., Sivers effect:
  - correlation between nucleon (transverse) polarization and quark transverse momentum
  - linked to orbital angular momentum
  - rigorous QCD prediction: breaking of universality of parton distributions by change of sign going from deep-inelastic scattering to Drell-Yan
transverse-momentum distributions

\[ f_1(x, k_T^2) \]

flavor-dependent tomographic maps in momentum space
transverse-momentum distributions

with spin: many possible configurations
TMDs - probabilistic interpretation

proton goes out of the screen / photon goes into the screen

\[ f_{1T} = \]
\[ h_{1L} = \]
\[ g_{1T} = \]
\[ h_{1T} = \]

nucleon with transverse or longitudinal spin
parton with transverse or longitudinal spin
parton transverse momentum

[courtesy of A. Bacchetta, Pavia]
TMDs - probabilistic interpretation

proton goes out of the screen / photon goes into the screen

parton with transverse or longitudinal spin

parton transverse momentum

\[ f_1 = \]

\[ g_1 = \]

\[ h_1 = \]

[courtesy of A. Bacchetta, Pavia]
inclusive DIS (one-photon exchange)

DIS … deep-inelastic scattering
inclusive DIS (one-photon exchange)

\[
\frac{d^2 \sigma(s, S)}{dx \, dQ^2} = \frac{2\pi \alpha^2 y^2}{Q^6} \mathbf{L}_{\mu\nu}(s) \mathbf{W}^{\mu\nu}(S)
\]

Lepton Tensor

Hadron Tensor

parametrized in terms of Structure Functions

\[
\frac{d^3 \sigma}{dx dy d\phi} \propto \frac{y}{2} F_1(x, Q^2) + \frac{1 - y - \gamma^2 y^2 / 4}{2xy} F_2(x, Q^2)
\]

\[-S_l S_N \cos \alpha \left[ \left( 1 - \frac{y}{2} - \frac{\gamma^2 y^2}{4} \right) g_1(x, Q^2) - \frac{\gamma^2 y}{2} g_2(x, Q^2) \right]
\]

\[+ S_l S_N \sin \alpha \cos \phi \gamma \sqrt{1 - y - \frac{\gamma^2 y^2}{4}} \left( \frac{y}{2} g_1(x, Q^2) + g_2(x, Q^2) \right)\]
TMDs - probabilistic interpretation

proton goes out of the screen / photon goes into the screen

\[ f_{1T} = \]

\[ h_{1} = \]

\[ g_{1T} = \]

\[ h_{1L} = \]

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[Courtesy of A. Bacchetta, Pavia]
quark polarimetry

- unpolarized quarks: easy - “just” hit them
- longitudinally polarized quarks: use polarized beam
quark polarimetry

- unpolarized quarks: easy - “just” hit them
- longitudinally polarized quarks: use polarized beam
- transversely polarized quarks: need final-state polarimetry, e.g.
semi-inclusive DIS

\[(E, p) \rightarrow (E', p')\]

e \rightarrow q \gamma^* \rightarrow h, K, \pi, u, d, u

parton distributions

fragmentation functions
probing TMDs in semi-inclusive DIS

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in SIDIS\(^*)\) couple PDFs to:

\((E, p)\) to \((E', p')\) in semi-inclusive DIS with unpolarized final state

\(^*)\) semi-inclusive DIS with unpolarized final state
probing TMDs in semi-inclusive DIS

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in SIDIS*) couple PDFs to:

Collins FF: $H_{1}^{\perp}, q \rightarrow h$

*) semi-inclusive DIS with unpolarized final state
probing TMDs in semi-inclusive DIS

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in SIDIS\(^{\*)} \) couple PDFs to:

**Collins FF:** $H_{1U}^{\perp}, q \rightarrow h$

**Ordinary FF:** $D_{1}^{q \rightarrow h}$

\(^{\*)} \) semi-inclusive DIS with unpolarized final state
probing TMDs in semi-inclusive DIS

in SIDIS*) couple PDFs to:

Collins FF: $H^{1\perp, q \rightarrow h}$

ordinary FF: $D^{q \rightarrow h}$

gives rise to characteristic azimuthal dependences

*) semi-inclusive DIS with unpolarized final state
single-hadron electroproduction (ep → ePX)

\[ d\sigma = d\sigma^{0}_{UU} + \cos 2\phi \, d\sigma^{1}_{UU} + \frac{1}{Q} \cos \phi \, d\sigma^{2}_{UU} + \lambda_e \frac{1}{Q} \sin \phi \, d\sigma^{3}_{LU} \]

\[ + S_L \left\{ \sin 2\phi \, d\sigma^{4}_{UL} + \frac{1}{Q} \sin \phi \, d\sigma^{5}_{UL} + \lambda_e \left[ d\sigma^{6}_{LL} + \frac{1}{Q} \cos \phi \, d\sigma^{7}_{LL} \right] \right\} \]

\[ + S_T \left\{ \sin(\phi - \phi_S) \, d\sigma^{8}_{UT} + \sin(\phi + \phi_S) \, d\sigma^{9}_{UT} + \sin(3\phi - \phi_S) \, d\sigma^{10}_{UT} \frac{1}{Q} \right. \]

\[ + \frac{1}{Q} \left( \sin(2\phi - \phi_S) \, d\sigma^{11}_{UT} + \sin \phi_S \, d\sigma^{12}_{UT} \right) \]

\[ + \lambda_e \left[ \cos(\phi - \phi_S) \, d\sigma^{13}_{LT} + \frac{1}{Q} \left( \cos \phi_S \, d\sigma^{14}_{LT} + \cos(2\phi - \phi_S) \, d\sigma^{15}_{LT} \right) \right] \}

Bacchetta et al., JHEP 0702 (2007) 093
single-hadron electroproduction (ep → e\(hX\))

\[
d\sigma = d\sigma_{UU}^0 + \cos 2\phi \ d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi \ d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi \ d\sigma_{LU}^3
\]

\[
+ S_L \left\{ \sin 2\phi \ d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi \ d\sigma_{UL}^5 + \lambda_e \left[ d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi \ d\sigma_{LL}^7 \right] \right\}
\]

\[
+ S_T \left\{ \sin(\phi - \phi_S) \ d\sigma_{UT}^8 + \sin(\phi + \phi_S) \ d\sigma_{UT}^9 + \sin(3\phi - \phi_S) \ d\sigma_{UT}^{10} \right.
\]

\[
+ \frac{1}{Q} \left( \sin(2\phi - \phi_S) \ d\sigma_{UT}^{11} + \sin \phi_S \ d\sigma_{UT}^{12} \right)
\]

\[
+ \lambda_e \left[ \cos(\phi - \phi_S) \ d\sigma_{LT}^{13} + \frac{1}{Q} \left( \cos \phi_S \ d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) \ d\sigma_{LT}^{15} \right) \right] \}
\]

Bacchetta et al., JHEP 0702 (2007) 093
single-hadron electroproduction (ep ⇆ eHX)

\[
d\sigma = d\sigma_{UU}^0 + \cos 2\phi d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi d\sigma_{LU}^3
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\[+ S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left( d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right) \right\} \]

\[+ S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \right. \]

\[\left. + \frac{1}{Q} \left( \sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12} \right) \right\} \]

\[+ \lambda_e \left[ \cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} \left( \cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15} \right) \right] \]

Bacchetta et al., JHEP 0702 (2007) 093
the rich world of fragmentation

<table>
<thead>
<tr>
<th>hadron pol.</th>
<th>U</th>
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<tbody>
<tr>
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<td>$H_1^\perp$</td>
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quark pol.
the rich world of fragmentation

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- unpolarized/spin-less hadrons
### the rich world of fragmentation

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- **unpolarized/spin-less hadrons**
- **polarized hadrons**

- 6 out of 8 require final-state polarimetry
- most accessible: hyperons (parity-violating decay), but
  - lower production rate
  - spin structure often dominated by strange quarks
- more involved: dihadron fragmentation functions
- clean process: $e^+e^-$ annihilation into hadron(s)
experimental data
The HERMES experiment (1995-2007)

27.5 GeV $e^+/e^-$ beam of HERA

... transversely polarized through Sokolov-Ternov effect

=> longitudinal polarization at HERMES by means of spin rotators
The HERMES experiment (1995-2007)

novel (pure) gas target:
- internal to HERA 27.6 GeV e± ring
- unpolarized (1H ... Xe)
- longitudinally polarized: 1H, 2H, 3He
- transversely polarized: 1H
transversely polarized quarks?

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- look at characteristic azimuthal dependence of single-hadron lepto-production cross section

$\vec{S}_\perp$, $\vec{p}_{\text{had}}$, $\phi$, $\phi_S$
transversely polarized quarks?

- look at characteristic azimuthal dependence of single-hadron lepto-production cross section
- in practice, reverse nucleon-polarization orientation and form spin asymmetries
transversely polarized quarks?

- look at characteristic azimuthal dependence of single-hadron lepto-production cross section
- in practice, reverse nucleon-polarization orientation and form spin asymmetries
- many of the systematics of polarization-averaged observables cancel (e.g., luminosity)
transversely polarized quarks?

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\[ \vec{p}_{\text{had}} \]

\[ \vec{S}_{\perp} \]

\[ \vec{k}' \]

\[ \vec{k} \]

\[ \phi_{S} \]

\[ \phi \]
transversely polarized quarks?

- transverse polarization of quarks leads to large effects!

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2005: First evidence from HERMES SIDIS on proton

Non-zero transversity
Non-zero Collins function

transversely polarized quarks?

- transverse polarization of quarks leads to large effects!
- opposite in sign for charged pions

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transversely polarized quarks?

- transverse polarization of quarks leads to large effects!
- opposite in sign for charged pions
- disfavored Collins FF large and opposite in sign to favored one

![Graph showing 2(sin(\phi + \phi_s))^2](image_url)

2005: First evidence from HERMES SIDIS on proton

Non-zero transversity
Non-zero Collins function

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[Airapetian et al., PLB 693 (2010) 11]
Collins effect for kaons and (anti) protons

\[ 2 \langle \sin(\phi_s + \phi_0) \rangle_{LT}^{K^\pm} \]
Collins effect for kaons and (anti) protons

positive Collins SSA amplitude for positive kaons (u-dominance)
Collins effect for kaons and (anti) protons

- positive Collins SSA amplitude for positive kaons (u-dominance)
- consistent with zero for negative kaons and (anti)protons

vanishing sea-quark transversity and baryon Collins effect?
Worm-Gear? Pretzelosity?

\[\begin{array}{|c|c|c|}
\hline
U & L & T \\
\hline
f_1 & h_1^+ & \\
\hline
f_{1L}^+ & h_{1L}^+ & \\
\hline
f_{1T}^+ & g_{1T}^+ & h_1, h_{1T}^+ \\
\hline
\end{array}\]

Fig. 5. Comparison of the measured analysing powers \(A_{\sin \phi} \) on the deuteron for \(\pi^+, \pi^0, \pi^-\) and \(K^+\) production with predictions from theoretical calculations in the chiral quark soliton model (\(\chi\)QSM, solid lines [22]), the quark–diquark model (QdQ, dashed lines [21]) and a perturbative QCD model (pQCD, dotted lines [21]). The shown curves refer to “approach 2” of the models in Ref. [21]. The error bars give the statistical uncertainties of the measurements, and the bands in the lower part of the panels show the systematic uncertainties of the measurements.

Fig. 6. The \(\sin 2\phi\) analysing powers \(A_{\sin 2\phi} \) for \(\pi^+, \pi^0\) and \(\pi^-\) (upper panel) and for \(K^+\) production (lower panel) on the deuteron. The error bars give the statistical uncertainties of the measurements. The systematic uncertainties for \(\pi^+\) and \(\pi^-\) are represented by the hatched band and those for \(\pi^0\) by the open band. The points for \(\pi^0\) and \(\pi^-\) are slightly shifted in \(x\) for better visibility. Included as curves are predictions from a transversity-related calculation in the chiral quark soliton model [22].

The data presented so far are evaluated in the semi-inclusive kinematic range \(0.2 < z < 0.7\). In Fig. 7, the \(z\)-dependencies of the single spin asymmetries \(A_{\sin \phi} \) on the proton and on the deuteron are shown up to \(z = 1\). The results on the proton have been obtained from experimental data taken with a longitudinally polarised hydrogen target as described in Ref. [2], neglecting the upper \(z < 0.7\). However, the mean experimental resolution in \(z\) is \(\Delta z = 0.02\) \((0.04)\) for charged (neutral) pions in the semi-inclusive regime and \(\Delta z = 0.07\) \((0.06)\) for \(z \to 1\). It has to be pointed out that the experimental data shown as open symbols in Fig. 7 have not been corrected for this variation in \(\Delta z\). Also, the results for charged pions have not been corrected for this.

\[\text{[PLB 562 (2003) 182-192]}\]
both consistent with zero power(s) of $P_{h\perp}$ (compared to, e.g., transversity\textcircled{C}ollins) but suppressed by one (two) power(s) of $P_{h\perp}$ compared to, e.g., $g_{1L}$, $h_{1L}$, $h_{1T}$.
signs of Boer-Mulders

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
U & L & T \\
\hline
$U_{f_{1}}$ & $g_{1L}$ & $h_{1L}^{+}$ \\
\hline
L & $f_{1T}$ & $h_{1}^{+}$, $h_{1T}^{+}$ \\
\hline
\end{tabular}
\end{table}

[Airapetian et al., PRD 87 (2013) 012010]

\[ e p \rightarrow e \pi X \]

\[ 2(\cos(2\phi_{h}^{u}))_{uu} \]

\[ P_{h_{\perp}} \text{ [GeV]} \]
signs of Boer-Mulders

<table>
<thead>
<tr>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>f_1</td>
<td></td>
<td>h_1^+</td>
</tr>
<tr>
<td>g_{1L}</td>
<td>h_{1L}^+</td>
<td></td>
</tr>
<tr>
<td>f_{1T}^+</td>
<td>g_{1T}</td>
<td>h_1, h_{1T}^+</td>
</tr>
</tbody>
</table>

[Airapetian et al., PRD 87 (2013) 012010]

- none-zero modulations!
signs of Boer-Mulders

- none-zero modulations!
- opposite sign for charged pions with larger magnitude for $\pi^-$

[Airapetian et al., PRD 87 (2013) 012010]
<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1$</td>
<td>$g_{1L}$</td>
<td>$h_{1L}^+$</td>
</tr>
<tr>
<td></td>
<td>$f_{1T}^-$</td>
<td>$g_{1T}^-$</td>
<td>$h_{1T}, h_{1T}^+$</td>
</tr>
</tbody>
</table>

- none-zero modulations!
- opposite sign for charged pions with larger magnitude for $\pi^-$
- intriguing behavior for kaons

[Airapetian et al., PRD 87 (2013) 012010]
signs of Boer-Mulders

- none-zero modulations!
- opposite sign for charged pions with larger magnitude for $\pi^-$
- intriguing behavior for kaons
- available also in fully differential binning, e.g., before projecting

[Airapetian et al., PRD 87 (2013) 012010]
chiral even

first direct evidence for worm-gear $g_{1T}$ on

- $^3$He target at JLab
- H target at HERMES
Sivers amplitudes for pions

\[ 2 \langle \sin (\phi - \phi_S) \rangle_{UT} = -\frac{\sum_q e_q^2 f_{1T}^q (x, p_T^2) \otimes W D_1^q (z, k_T^2)}{\sum_q e_q^2 f_{1T}^q (x, p_T^2) \otimes D_1^q (z, k_T^2)} \]
Sivers amplitudes for pions

\[ 2\langle \sin (\phi - \phi_S) \rangle_{UT} = -\frac{\sum_q e_q^2 f_{1T}^\perp (x, p_T^2) \otimes W D_q^q(z, k_T^2)}{\sum_q e_q^2 f_{1}^q (x, p_T^2) \otimes D_1^q(z, k_T^2)} \]

\( \pi^+ \) dominated by u-quark scattering:

\[ \sim -\frac{f_{1T}^\perp u(x, p_T^2) \otimes W D_1^{u \rightarrow \pi^+}(z, k_T^2)}{f_{1}^u(x, p_T^2) \otimes D_1^{u \rightarrow \pi^+}(z, k_T^2)} \]

\( \mathning u \)-quark Sivers DF < 0
Sivers amplitudes for pions

\[
2\langle \sin (\phi - \phi_S) \rangle_{UT} = -\frac{\sum_q e_q^2 f_{1T}^{\perp q}(x, p_T^2) \otimes W D_1^q(z, k_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)}
\]

\[\pi^+\] dominated by u-quark scattering:

\[
\sim - \frac{f_{1T}^{\perp u}(x, p_T^2) \otimes W D_1^{u \rightarrow \pi^+}(z, k_T^2)}{f_1^u(x, p_T^2) \otimes D_1^{u \rightarrow \pi^+}(z, k_T^2)}
\]

\[\uparrow\text{ u-quark Sivers DF } < 0\]

\[\downarrow\text{ d-quark Sivers DF } > 0\]

(cancellation for \(\pi^-\))
Sivers amplitudes for mesons

\[ 2 \langle \sin(\phi - \phi_S) \rangle_{UT} = -\frac{\sum_q e_q^2 f_{1T}^q(x, p_T^2) \otimes W D_1^q(z, k_T^2)}{\sum_q e_q^2 f_{1}^q(x, p_T^2) \otimes D_1^q(z, k_T^2)} \]

[larger amplitudes for positive kaons vs. pions]
Sivers amplitudes for baryons

\[ 2\langle \sin (\phi - \phi_S) \rangle_{UT} = -\frac{\sum_q e_q^2 f_{1T}^q (x, p_T^2) \otimes \mathcal{N} D_1^q (z, k_T^2)}{\sum_q e_q^2 f_{1T}^q (x, p_T^2) \otimes D_1^q (z, k_T^2)} \]

similar amplitudes for positive pions and protons ➡️ u-quark dominance

[Airapetian et al., PLB 693 (2010) 11]
$e^+e^-\text{ annihilation}$
fragmentation in $e^+e^-$ annihilation

Fig. 7. Quark–quark (a) and one of the quark–quark–gluon (b) correlators in tree-level diagrams for back-to-back jet production in electron–positron annihilation.

where the hermiticity properties of the various matrix elements have been used (see Section 7).


Also for 2-particle inclusive electron–positron annihilation we have a quite similar procedure. The calculation involves two soft fragmentation parts and the creation of a quark–antiquark pair. We will discuss only the case of creation from a (timelike) photon.

The handbag diagram is given in Fig. 7(a) and an example of a diagram involving an additional gluon in Fig. 7(b).

The calculation of this tensor in a diagrammatic expansion proceeds as in the case of leptoproduction and gives

$$W^\mu_\nu(q; P_1, S_1; P_2, S_2) = \int d^4 p d^4 k \delta^4(p + k - q) \left\{ Tr\left( \bar{\Delta}(p) \gamma^\mu \Delta(k) \gamma^\nu \right) - \int d^4 p_1 Tr\left( \gamma^\alpha / k + / p_1 + m(k + p_1)^2 - m^2 + i \epsilon \gamma^\nu \bar{\Delta}(p, p - p_1) \gamma^\mu \Delta(k) \Delta(\alpha) A(p, p - p_1) \right) - \int d^4 k_1 Tr\left( \gamma^\nu / p - / k_1 + m(p + k_1)^2 - m^2 - i \epsilon \gamma^\alpha \bar{\Delta}(p) \gamma^\mu \Delta(k_1 - k_1, k) \right) \right\} + \cdots \right.$$
fragmentation in $e^+e^-$ annihilation

- single-inclusive hadron production, $e^+e^- \rightarrow hX$
- $D_1$ fragmentation fctn.
- $D_1T^\perp$ spontaneous transv. pol.
fragmentation in $e^+e^-$ annihilation

- single-inclusive hadron production, $e^+e^- \rightarrow hX$
- $D_1$ fragmentation fctn.
- $D_1T^\perp$ spontaneous transv. pol.
- inclusive “back-to-back” hadron pairs, $e^+e^- \rightarrow h_1h_2X$
- product of FFs
- flavor, transverse-momentum, and/or polarization tagging

Thrust (axis):

$$T \equiv \max_{\hat{n}} \frac{\sum_h |P_{h,CMS} \cdot \hat{n}|}{\sum_h |P_{h,CMS}|}$$

\[ e^+ \rightarrow P_{h_1}, \quad e^- \rightarrow P_{h_2} \]
fragmentation in $e^+e^-$ annihilation

- single-inclusive hadron production, $e^+e^- \rightarrow hX$
- $D_1$ fragmentation fctn.
- $D_{1T}^\perp$ spontaneous transv. pol.
- inclusive “back-to-back” hadron pairs, $e^+e^- \rightarrow h_1h_2X$
- product of FFs
- flavor, transverse-momentum, and/or polarization tagging
- inclusive same-hemisphere hadron pairs, $e^+e^- \rightarrow h_1h_2X$
- dihadron fragmentation

$\text{Thrust (axis):}$

$$T = \frac{\sum_h |\mathbf{p}_{h}^{\text{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{p}_{h}^{\text{CMS}}|}$$
e^+e^- annihilation at BESIII, BaBar & Belle

- BaBar & Belle: asymmetric beam-energy e^+e^- collider near/at Y(4S) resonance (10.58 GeV)
- BESIII: symmetric collider with E_e=1...2.4 GeV
e^+e^- annihilation at BESIII, BaBar & Belle

- **BaBar & Belle:** asymmetric beam-energy e^+e^- collider near/at Y(4S) resonance (10.58 GeV)
- **BESIII:** symmetric collider with E_e=1...2.4 GeV
- integrated luminosities:

<table>
<thead>
<tr>
<th></th>
<th>Y(4S) on resonance</th>
<th>Y(4S) off resonance</th>
<th>other</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>424.2 fb^{-1}</td>
<td>43.9 fb^{-1}</td>
<td></td>
</tr>
<tr>
<td>Belle</td>
<td>(140+571) fb^{-1}</td>
<td>(15.6+73.8) fb^{-1}</td>
<td></td>
</tr>
<tr>
<td>BESIII</td>
<td></td>
<td></td>
<td>~62 pb^{-1} @3.65 GeV *)</td>
</tr>
</tbody>
</table>

*) used for the Collins analysis presented here
single-hadron production

- before 2013: lack of precision data at (moderately) high $z$ and at low $\sqrt{s}$
- limits analysis of evolution and gluon fragmentation
- limited information in kinematic region often used in semi-inclusive DIS

---

```latex
\textbf{Fig. 19.2:} Fragmentation functions in lower values. This can be seen from Fig. 19.2 where a large amo
```

---

\[ F(x) = \frac{d\sigma}{dx} \times c(\sqrt{s}) \]

---

\[ \text{where the leading contribution is of order } z^{(1-2 \ln s)} \]
single-hadron production

- before 2013: lack of precision data at (moderately) high $z$ and at low $\sqrt{s}$
- limits analysis of evolution and gluon fragmentation
- limited information in kinematic region often used in semi-inclusive DIS

now, results available from BaBar and Belle:

- BaBar Collaboration, Phys. Rev. D88 (2013) 032011: $\pi^\pm$, $K^\pm$, $p+p$
- Belle Collaboration, Phys. Rev. Lett. 111 (2013) 062002: $\pi^\pm$, $K^\pm$
- Belle Collaboration, Phys. Rev. D92 (2015) 092007: $\pi^\pm$, $K^\pm$, $p+p$
single-hadron production

- very precise data for charged pions and kaons
- Belle data available up to very large z (z<0.98)
- included in recent DEHSS fits
- slight tension at low-z for BaBar and high-z for Belle
single-hadron production

- very precise data for charged pions and kaons
- Belle data available up to very large z (z<0.98)
- included in recent DEHSS fits [e.g. PRD 91, 014035 (2015)]
- Belle radiative corrections **undone** in FF fits

In the case of the BELLE experiment we multiply all data points by a factor \( 1/c \), with \( c = 0.65 \) for charged pions and kaons [69] and with \( c \) a function of \( z \) for protons/antiprotons [53]. This correction is required in order to treat the BELLE data consistently with all the other SIA measurements included in NNFF1.0. The reason is that a kinematic cut on radiative photon events was applied to the BELLE data sample in the original analysis instead of unfolding the radiative QED effects. Specifically, the energy scales...
single-hadron production

- very precise data for charged pions and kaons
- Belle data available up to very large z (z<0.98)
- included in recent DEHSS fits [e.g. PRD 91, 014035 (2015)]
- Belle radiative corrections **undone** in FF fits
- data for protons & anti-protons
  - not (yet) included in DEHSS, but in NNFF 1.0 [EPJC 77 (2017) 516]
  - similar z dependence as pions
  - about ~½ of pion cross sections

**Note:**
- The diagrams show comparisons between different models and experimental data for single hadron production. The axes and scales are specific to the context of the plots, focusing on cross sections and distributions. The data points and model predictions are crucial for understanding the agreement or discrepancies in hadron production at high energies.
inclusive hyperon production

- $\Lambda$ production reasonably well described by Pythia
- less satisfactory for heavier hyperons
- fails to describe $\Omega^-$ production

M. Niiyama et al., PRD 97 (2018) 072005
hadron-pair production

- Single-hadron production has low discriminating power for parton flavor.
- Can use 2nd hadron in opposite hemisphere to “tag” flavor.
- Mainly sensitive to product of single-hadron FFs.
- If hadrons in same hemisphere: dihadron fragmentation.

- Opens the question of defining hemispheres.
no hemisphere selection
hadron-pair production


no hemisphere selection

warning: a factor of two missing for unlike-hadron pairs

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no hemisphere selection

hadron-pair production

FIG. 12 (color online). Differential cross sections for

\[ \frac{d^2\sigma}{dz_1 dz_2} \] for various center-of-mass energies. The data are shown in different colors for various hadron pairs: ππ, πK, K\bar{K}, and K\bar{K}.

The graph shows that the cross sections increase with increasing fractional energies. This is indeed the case as can be seen in Fig. 85.
no hemisphere selection

FIG. 12 (color online). Differential cross sections for fractional energies of the two final-state hadrons. If a significant fraction of the energy is transferred to a single hadron, the other one is significantly softer.

- For $0.20 < z_1 < 0.25$, the cross sections decrease as $z_2$ increases, with data points closer to the $z_2 = 0.5$ line.
- For $0.40 < z_1 < 0.45$, the cross sections show a similar trend, with a slight increase near $z_2 = 0.5$.
- For $0.60 < z_1 < 0.65$, the decrease is more pronounced, with a sharp drop near $z_2 = 0.5$.
- For $0.80 < z_1 < 0.85$, the decrease is similar to the previous intervals, with a slight increase near $z_2 = 0.5$.

The data points are color-coded for different hadron combinations: $\pi^+\pi^-$, $\pi^-K^+$, $\pi^+K^-$, and $K^-K^+$. The cross sections are shown on a log-log scale for each $z_1$ interval.
**hadron-pairs: topology comparison**

- any hemisphere vs. opposite- & same-hemisphere pairs
- same-hemisphere pairs with kinematic limit at $z_1 = z_2 = 0.5$

![Graph showing differential cross sections and ratios for main hadron combinations, with different color and style for same-hemisphere and opposite-hemisphere data, comparing to those without hemisphere assignment.](image)

FIG. 9. Differential cross sections for $\pi^+\pi^-$ (black circles) and $\pi^+\pi^+\pi^+$ (blue squares) as a function of $m_{\pi\pi}$ for the indicated $z$ bins. The error boxes represent the systematic uncertainties. Top panel: linear representation of cross sections; bottom panel: logarithmic representation. The vertical green dashed line corresponds to the kinematic limit. An overall 1.6% scale uncertainty is not shown.

same-hemisphere data: $M_{h_1h_2}$ dependence

unlike-sign hadron pairs

like-sign hadron pairs

$\pi^+\pi^-$ Data

$\pi^+\pi^+$ Data

same-hemisphere data: $M_{h_1 h_2}$ dependence

 Unlike-sign hadron pairs

 Like-sign hadron pairs

\[ \pi^+\pi^- \text{ Data} \]

\[ \pi^+\pi^+ \text{ Data} \]

$T > 0.8$
$z_{1,2} > 0.1$

- Unlike-sign pairs with clear decay and resonance structure: $K_s, \rho^0 \ldots$
- Like-sign pairs with much smoother and smaller cross sections

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polarization
hadron pairs: angular correlations

- **angular correlations** between nearly back-to-back hadrons used to **tag** transverse quark **polarization** -> Collins fragmentation fct.

- RF0: one hadron as reference axis  -> \( \cos(2\phi_0) \) modulation
- RF12: thrust (or similar) axis  -> \( \cos(\phi_1 + \phi_2) \) modulation

- RF0 and RF12: different convolutions over transverse momenta
- debatable: MC used to “correct” thrust axis to \( q\bar{q} \) axis
hadron pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., MC)

![Graph](Phys. Rev. D90 (2014) 052003)
hadron pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., MC)
- construct double ratio of normalized-yield distributions $R_{12}$, e.g. unlike-/like-sign:

$$ \frac{R_{12}^U}{R_{12}^L} \simeq 1 + \left\langle \frac{\sin^2 \theta_{th}}{1 + \cos^2 \theta_{th}} \right\rangle G^U \cos(\phi_1 + \phi_2) $$

$$ \simeq 1 + \left\langle \frac{\sin^2 \theta_{th}}{1 + \cos^2 \theta_{th}} \right\rangle \{ G^U - G^L \} \cos(\phi_1 + \phi_2) $$

- suppresses flavor-independent sources of modulations
- $G^U/L$ specific combinations of FFs
- remaining MC asymmetries: -> systematics
Collins asymmetries (RF0)


- significant asymmetries rising with $z$

- used for first transversity and Collins FF extractions
Collins asymmetries (RF0)


- BaBar results [PRD 90 (2014) 052003] consistent with Belle

- BaBar results [PRD 90 (2014) 052003] consistent with Belle
**Collins asymmetries (RF0)**

**BaBar results** [PRD 90 (2014) 052003] consistent with Belle

**BESIII** [PRL 116 (2016) 042001] (at smaller s) consistent with TMD evolution [Z.-B. Kang et al., PRD 93 (2016) 014009]

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**PHYSICAL REVIEW D**

**FIG. 2. Double ratio**

- **A UL**
- **A UC**

**Table**

the HERMES experiment

the UL (up triangles) and UC (down triangles) ratios are reported, with statistical error bars and systematic uncertainties represented by

- **J. P. LEES**
- **B. Collins asymmetries vs transverse momenta**

---

*Image*
Collins asymmetries - going further

- even larger effects seen for kaon pairs

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Collins asymmetries - going further

- even larger effects seen for kaon pairs
- \( p_T \) dependence for pions

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polarizing fragmentation function

- polarization normal to production plane, i.e. $\propto (\vec{q} \times \vec{P}_\Lambda)$
  [note that the sign got reversed in the drawing]

\[ \hat{n} \text{ is perpendicular to the } \Lambda \text{ production plane.} \]

- reference axis to define transverse momentum:
  - “thrust frame” - use thrust axis
  - “hadron frame” - use momentum direction of “back-to-back” hadron
polarizing fragmentation function

- flavor tagging through hadrons in opposite hemisphere:

- large-$z_h$ hadrons tag quark flavor more efficiently

  ➡️ enlarges differences between oppositely charged hadrons
instead of summarizing results ...
a QCD laboratory

DIS

\[ P_h \]

\[ \Phi \]

\[ \Delta \]

\[ P \]

\[ p \]

\[ q \]

\[ k \]

\[ \text{Drell-Yan} \]

\[ P_{B} \]

\[ P_{A} \]

\[ k \]

\[ q \]

\[ \Phi \]

\[ P_{B} \]

\[ P_{B} \]

\[ P_{A} \]

\[ P_{A} \]

\[ \text{e}^+ \text{e}^- \]

\[ P_{1} \]

\[ P_{1} \]

\[ P_{2} \]

\[ P_{2} \]

\[ k \]

\[ q \]

\[ \bar{\Delta} \]
DIS the hadron momentum defines the lightcone direction.

hadron structure (distribution functions)

Drell-Yan

in Figs. 1(a) and 2(a) only involve quark–quark matrix elements. In

operator combination with a gauge link can be expanded into a tower of local twist-two

The situation in SIDIS (Fig. 2), discussed in Section 3, differs in a subtle way from

involves now two soft distribution parts and annihilation of a quark–antiquark pair into

5. The Drell–Yan cross sections

For Drell–Yan, one has a similar treatment as for leptoproduction. The calculation

jet production in electron–positron annihilation.

also for 2-particle inclusive electron–positron annihilation we have a quite similar

procedure. The calculation involves two soft fragmentation parts and the creation of a

additional gluon in Fig. 7(b).
a QCD laboratory

hadronization
(fragmentation functions)

DIS

e^+e^-

Drell-Yan
a QCD laboratory

- data from HERMES, JLab and COMPASS; planned for future EIC
- convolutes parton distribution (Φ) and fragmentation (Δ) functions Φ⊗Δ
- need fragmentation function to extract distribution functions

Drell-Yan
data from HERMES, JLab and COMPASS; planned for future EIC
convolutes parton distribution ($\Phi$) and fragmentation ($\Delta$) functions $\Phi \otimes \Delta$
need fragmentation function to extract distribution functions

ideal place to study hadronization
convolutes parton fragmentation functions $\Delta \otimes \Delta$
wealth of existing data from BELLE/ BaBar & BESIII and more to come (especially Belle2)
a QCD laboratory

- data from HERMES, JLab and COMPASS; planned for future EIC
- convolutes parton distribution (Φ) and fragmentation (Δ) functions Φ⊗Δ
- need fragmentation function to extract distribution functions
- testing ground for universality of TMDs
- convolutes parton distribution functions Φ⊗Φ
- measurable at COMPASS, RHIC, Fermilab and LHC
- ideal place to study hadronization
- convolutes parton fragmentation functions Δ⊗Δ
- wealth of existing data from BELLE/ BaBar & BESIII and more to come (especially Belle2)

data from HERMES, JLab and COMPASS; planned for future EIC

convolutes parton distribution (Φ) and fragmentation (Δ) functions Φ⊗Δ

need fragmentation function to extract distribution functions

testing ground for universality of TMDs

convolutes parton distribution functions Φ⊗Φ

measurable at COMPASS, RHIC, Fermilab and LHC

ideal place to study hadronization

convolutes parton fragmentation functions Δ⊗Δ

wealth of existing data from BELLE/ BaBar & BESIII and more to come (especially Belle2)
backup
Process dependence

simple QED example

DIS: attractive

Drell-Yan: repulsive
Process dependence

simple QED example

DIS: attractive

Drell-Yan: repulsive

add color: QCD

result: \( Sivers|_{\text{DIS}} = - Sivers|_{\text{DY}} \)
Process dependence

simple QED example

DIS: attractive
Drell-Yan: repulsive

add color: QCD

rigorous QCD prediction not tested!! - need Drell-Yan data

result: Sivers|_{DIS} = - Sivers|_{DY}