Recent Hermes results for SSAs and DSAs

Luciano L. Pappalardo

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The phase-space distribution of partons

The full phase-space distribution of the partons encoded in the Wigner function $W(x, p_T, r)$.
The phase-space distribution of partons

The full phase-space distribution of the partons encoded in the Wigner function

...but $\Delta x \Delta p \geq \frac{\hbar}{2} \rightarrow$ no simultaneous knowledge of momentum and position cannot be directly accessed experimentally $\rightarrow$ integrated quantities

\[
\begin{align*}
W(x, p_T, r) & \quad \text{TMDs} \\
\int d^3r & \quad \int d^2p_T \\
f(x, p_T) & \quad H(x, \xi, t) \\
\text{GPDs}
\end{align*}
\]
The non-collinear structure of the nucleon

The full phase-space distribution of the partons encoded in the Wigner function

...but \( \Delta x \Delta p \geq \frac{\hbar}{2} \) → no simultaneous knowledge of momentum and position cannot be directly accessed experimentally \( \rightarrow \) integrated quantities

- TMDs depend on \( x \) and \( p_T \)
- Describe correlations between \( p_T \) and quark or nucleon spin (spin-orbit correlations)
- Provide a 3-dim picture of the nucleon in momentum space (nucleon tomography)

\[
\int d^3 r \quad W(x, p_T, r)
\]

\[
\int d^2 p_T \quad H(x, \xi, t)
\]

- TMDs
- \( f(x, p_T) \)
- GPDs

\( \bar{p} \rightarrow q \rightarrow p_T \)

\( \bar{p} = xP \)

\( \text{momentum} \)
\( \text{helicity} \)
\( \text{Boer-Mulders} \)

\( \Delta x \Delta p \geq \frac{\hbar}{2} \) → no simultaneous knowledge of momentum and position cannot be directly accessed experimentally \( \rightarrow \) integrated quantities

\[
\text{Sivers} \quad \text{worm-gears}
\]

\[
\text{transversity} \quad \text{pretzelosity}
\]

L.L. Pappalardo – Structure of Nucleons and Nuclei – Como – June 10-14 2013
The non-collinear structure of the nucleon

Mostly investigated in **SIDIS**: detection of transverse momentum of produced hadrons gives access to $p_T$

- TMDs depend on $x$ and $p_T$
- Describe correlations between $p_T$ and quark or nucleon spin (spin-orbit correlations)
- Provide a **3-dim picture** of the nucleon in momentum space (nucleon tomography)
The SIDIS cross-section

\[
\frac{d\sigma^h}{dx \, dy \, d\phi_S \, dz \, d\phi \, dP_{h\perp}^2} = \frac{\alpha^2}{x y Q^2} \frac{y^2}{2 (1 - \epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right)
\]

\[
\left\{ \begin{array}{l}
F_{UU,T} + \epsilon F_{UU,L} \\
\sqrt{2\epsilon (1 + \epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \\
+ \lambda_l \left[ \sqrt{2\epsilon (1 - \epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\
+ S_L \left[ \sqrt{2\epsilon (1 + \epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\
+ S_L \lambda_l \left[ \sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2\epsilon (1 - \epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\
+ S_T \left[ \sin(\phi - \phi_s) \left( F_{UT,T}^{\sin(\phi - \phi_s)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_s)} \right) \\
+ \epsilon \sin(\phi + \phi_s) F_{UT}^{\sin(\phi + \phi_s)} + \epsilon \sin(3\phi - \phi_s) F_{UT}^{\sin(3\phi - \phi_s)} \\
+ \sqrt{2\epsilon (1 + \epsilon)} \sin(\phi_s) F_{UT}^{\sin(\phi_s)} \\
+ \sqrt{2\epsilon (1 + \epsilon)} \sin(2\phi - \phi_s) F_{UT}^{\sin(2\phi - \phi_s)} \right] \\
+ S_T \lambda_l \left[ \sqrt{1 - \epsilon^2} \cos(\phi - \phi_s) F_{LT}^{\cos(\phi - \phi_s)} \\
+ \sqrt{2\epsilon (1 - \epsilon)} \cos(\phi_s) F_{LT}^{\cos(\phi_s)} \\
+ \sqrt{2\epsilon (1 - \epsilon)} \cos(2\phi - \phi_s) F_{LT}^{\cos(2\phi - \phi_s)} \right] \end{array} \right\}
\]

\[
F_{XY,Z} = F_{XY,Z}(x, y, z)
\]
The SIDIS cross-section

\[ \frac{d\sigma^h}{dx \, dy \, d\phi \, d\rho \, dq^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \]

\[ \left\{ \begin{array}{l}
F_{UU,T} + \epsilon F_{UU,L} \\
+ \sqrt{2\epsilon(1+\epsilon)} \cos(\phi)F_{UU}^{\text{cos}}(\phi) + \epsilon \cos(2\phi)F_{UU}^{\text{cos}}(2\phi) \\
+ \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi)F_{LU}^{\text{sin}}(\phi) \right] \\
+ S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi)F_{UL}^{\text{sin}}(\phi) + \epsilon \sin(2\phi)F_{UL}^{\text{sin}}(2\phi) \right] \\
+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi)F_{LL}^{\text{cos}}(\phi) \right] \\
+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\text{sin}}(\phi - \phi_S) + \epsilon F_{UT,L}^{\text{sin}}(\phi - \phi_S) \right) \\
+ \epsilon \sin(\phi + \phi_S)F_{UT}^{\text{sin}}(\phi + \phi_S) + \epsilon \sin(3\phi - \phi_S)F_{UT}^{\text{sin}}(3\phi - \phi_S) \\
+ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S)F_{UT}^{\text{sin}}(\phi_S) \right] \\
+ \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S)F_{UT}^{\text{sin}}(2\phi - \phi_S) \right] \\
+ S_T \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S)F_{LT}^{\text{cos}}(\phi - \phi_S) \\
+ \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S)F_{LT}^{\text{cos}}(\phi_S) \\
+ \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S)F_{LT}^{\text{cos}}(2\phi - \phi_S) \right] \right\} \]

unpolarized

beam polarization

target polarization

beam and target polarization

target polarization

beam polarization

virtual photon polarization

\[ F_{XY,Z} = F_{XY,Z}(x, y, z) \]
Selected leading-twist 1-hadron SIDIS results
**Boer–Mulders function**

\[
\frac{d\sigma^h}{dxdydzd\phi dP^2_{h,1}} = \frac{\alpha^2 \gamma^2}{xyQ^2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \}
\]

\[
\left\{ \begin{array}{l}
F_{UU,T} + \epsilon F_{UU,L} \\
\sqrt{2\epsilon (1 + \epsilon)} \cos (\phi) F_{UU}^{\cos (\phi)} + \epsilon \cos (2\phi) F_{UU}^{\cos (2\phi)} \\
\lambda_l \left[ \sqrt{2\epsilon (1 - \epsilon)} \sin (\phi) F_{LU}^{\sin (\phi)} \right] \\
S_L \left[ \sqrt{2\epsilon (1 + \epsilon)} \sin (\phi) F_{UL}^{\sin (\phi)} + \epsilon \sin (2\phi) F_{UL}^{\sin (2\phi)} \right] \\
S_L \lambda_l \left[ \sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2\epsilon (1 - \epsilon)} \cos (\phi) F_{UL}^{\cos (\phi)} \right] \\
S_T \left[ \sin (\phi - \phi S) \left( F_{UT,T}^{\sin (\phi - \phi S)} + \epsilon F_{UT,L}^{\sin (\phi - \phi S)} \right) \\
+ \epsilon \sin (\phi - \phi S) F_{UT}^{\sin (\phi + \phi S)} + \epsilon \sin (3\phi - \phi S) F_{UT}^{\sin (3\phi - \phi S)} \\
+ \sqrt{2\epsilon (1 + \epsilon)} \sin (\phi S) F_{UT}^{\sin (\phi S)} \\
+ \sqrt{2\epsilon (1 + \epsilon)} \sin (2\phi - \phi S) F_{UT}^{\sin (2\phi - \phi S)} \right] \\
S_T \lambda_l \left[ \sqrt{1 - \epsilon^2} \cos (\phi - \phi S) F_{LT}^{\cos (\phi - \phi S)} \\
+ \sqrt{2\epsilon (1 - \epsilon)} \cos (\phi S) F_{LT}^{\cos (\phi S)} \\
+ \sqrt{2\epsilon (1 - \epsilon)} \cos (2\phi - \phi S) F_{LT}^{\cos (2\phi - \phi S)} \right] \}
\]

\[
F_{UU}^{\cos 2\phi_h} = C \left[ -\frac{2(\hat{n} \cdot k_T)(\hat{n} \cdot p_T) - k_T \cdot p_T}{MM_h} h_1^{\perp} H_1^{\perp} \right]
\]

Describes correlation between quark transverse momentum and transverse spin in unpolarized nucleon

### Distribution Functions

<table>
<thead>
<tr>
<th>quark</th>
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<tr>
<td>U</td>
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### Fragmentation Functions

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<tr>
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<td>$D_1$</td>
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<tr>
<td>h</td>
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<td>$H_1^{\perp}$</td>
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</table>
The \( \cos 2\phi \) amplitudes \( \propto h_1^\perp(x, p_T^2) \otimes H_1^\perp(z, k_T^2) \)

- Amplitudes are significant → clear evidence of BM effect
- Similar results for H & D indicate \( h_1^{u, u} \approx h_1^{d, d} \)
- Opposite sign for \( \pi^+/\pi^- \) consistent with opposite signs of fav/unfav Collins
The cos2φ amplitudes \( \propto h_1^\perp (x, p_T^2) \otimes H_1^\perp (z, k_T^2) \)

- Amplitudes are significant → clear evidence of BM effect
- similar results for H & D indicate \( h_1^{\perp, u} \approx h_1^{\perp, d} \)
- Opposite sign for \( \pi^+ / \pi^- \)
  consistent with opposite signs of fav/unfav Collins

\[ \begin{array}{c}
\text{negative} \\
\text{positive}
\end{array} \]

\[ \begin{array}{c}
\text{Large and negative} \\
\text{Large and negative}
\end{array} \]

- \( K^+ / K^- \) amplitudes are larger than for pions , have different kinematic dependencies than pions and have same sign
- different role of Collins FF for pions and kaons?
- Significant contribution from scattering off strange quarks?

Analysis multi-dimensional in x, y, z, and Pt
Create your own projections of results through: http://www-hermes.desy.de/cosnphi/
Transversity

\[
\frac{d\sigma^h}{dx\,dy\,d\phi\,dS\,dz\,d\phi\,dP^2_{h\perp}} = \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1 - \epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right)
\]

\[
\begin{align*}
\{ & F_{UU,T} + \epsilon F_{UU,L} \\
& + \sqrt{2\epsilon(1 + \epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \\
& + \lambda_t \{ \sqrt{2\epsilon(1 - \epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \\
& + S_L \{ \sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2\epsilon(1 - \epsilon)} \cos(\phi) F_{UL}^{\cos(\phi)} \\
& + S_T \{ \sin(\phi - \phi_S)(F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)}) \\
& + \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \\
& + \sqrt{2\epsilon(1 + \epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\
& + \sqrt{2\epsilon(1 + \epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \}
\}
\end{align*}
\]

Describes probability to find transversely polarized quarks in a transversely polarized nucleon

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**Distribution Functions**

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<td>$h^+_1$</td>
<td>$-\circ$</td>
</tr>
<tr>
<td>L</td>
<td>$g_1$</td>
<td>$h^-_{1L}$</td>
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<td>T</td>
<td>$f^+_1$</td>
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**Fragmentation Functions**

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Collins amplitudes $\propto h_1(x, p_T^2) \otimes H_1^\perp(z, k_T^2)$

Consistent with Belle/BaBar measurements in $e^+e^-$

$H_{1,unf}^fav(z) \approx -H_{1,fav}^unf(z)$

Consistent with Belle/BaBar measurements in $e^+e^-$

- $e^+e^- \rightarrow \pi^+_{\text{jet}1} \pi^-_{\text{jet}2} X$

\[ \begin{align*}
\left[ u \rightarrow \pi^- \quad \frac{d \rightarrow \pi^+}{d \rightarrow \pi^-} \right]
\end{align*} \]

\[ \begin{align*}
2 (\sin(\phi - \phi_0))_{UT}^\pi^+ & \\
2 (\sin(\phi - \phi_0))_{UT}^{\pi^0} & \\
2 (\sin(\phi - \phi_0))_{UT}^{\pi^-} & \\
2 (\sin(\phi - \phi_0))_{UT}^{K^+} & \\
2 (\sin(\phi - \phi_0))_{UT}^{K^-} & \\
\end{align*} \]

- positive
- consistent with zero (isospin-symmetry)
- large and negative!
- significantly positive
- consistent with zero

\[ \begin{align*}
\text{Anselmino et al. Phys. Rev. D 75 (2007)}
\end{align*} \]

\[ \begin{align*}
\end{align*} \]
Sivers function

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi dP^2_{h\perp}} = \frac{\alpha^2 y^2}{x y Q^2 2 (1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right)$$

\[
\begin{align*}
F_{UU,T} + \epsilon F_{UU,L} \\
+ \sqrt{2\epsilon (1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \\
+ \lambda_l \sqrt{2\epsilon (1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \\
+ S_L \left[ \sqrt{2\epsilon (1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\
+ S_L \lambda_l \left[ \sqrt{1-\epsilon_2} F_{LL} + \sqrt{2\epsilon (1-\epsilon)} \cos(\phi) F_{UL}^{\cos(\phi)} \right] \\
+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) \\
+ \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \\
+ \sqrt{2\epsilon (1+\epsilon)} \cos(\phi_S) F_{UT}^{\cos(\phi_S)} \\
+ \sqrt{2\epsilon (1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right] \\
+ S_T \lambda_l \left[ \sqrt{1-\epsilon_2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \\
+ \sqrt{2\epsilon (1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\
+ \sqrt{2\epsilon (1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right]
\end{align*}
\]

Describes correlation between quark transverse momentum and nucleon transverse polarization.

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<tr>
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<td>$g_1$</td>
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</table>
Sivers amplitudes $\propto f_{1T}^\perp(x, p_T^2) \otimes D_1(z, k_T^2)$

consistent with Sivers func. of opposite sign for u and d quarks

$2(\sin(\phi_B)_{\pi^+}, \pi^0, \pi^-)$

significantly positive

slightly positive

(isospin-symmetry)

consistent with zero


Sivers amplitudes $\propto f_{1T}(x, p_T^2) \otimes D_1(z, k_T^2)$

- **significantly positive**
- **slightly positive** (isospin-symmetry)
- **consistent with zero**
- **Larger than $\pi^+$!!**

- role of sea quarks?

- Higher-twist contrib for $K^+$?

consistent with Sivers func. of opposite sign for $u$ and $d$ quarks


- Lower twist contrib for $K^+$

Worm-gear $g^\perp_{1T}$

\[
\frac{d\sigma^h}{dx dy d\phi_S dz d\phi dP^2_{h\perp}} = \frac{\alpha^2 y^2}{x y Q^2 2(1 - \epsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left[ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1 + \epsilon)} \cos(\phi)F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi)F_{UU}^{\cos(2\phi)} \right] \]

\[
+ \lambda_l \left[ \sqrt{2\epsilon(1 - \epsilon)} \sin(\phi)F_{UL}^{\sin(\phi)} \right]
\]

\[
+ S_L \left[ \sqrt{2\epsilon(1 - \epsilon)} \sin(\phi)F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi)F_{UL}^{\sin(2\phi)} \right]
\]

\[
+ S_L \lambda_l \left[ \sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2\epsilon(1 - \epsilon)} \cos(\phi)F_{LL}^{\cos(\phi)} \right]
\]

\[
+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UU,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UU,L}^{\sin(\phi - \phi_S)} \right) + \epsilon \sin(\phi + \phi_S)F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S)F_{UT}^{\sin(3\phi - \phi_S)} \right.
\]

\[
+ \sqrt{2\epsilon(1 + \epsilon)} \sin(\phi_S)F_{UT}^{\sin(\phi_S)} + \sqrt{2\epsilon(1 + \epsilon)} \sin(2\phi - \phi_S)F_{UT}^{\sin(2\phi - \phi_S)} \left. \right]
\]

\[
+ S_T \lambda_l \left[ \sqrt{1 - \epsilon^2} \cos(\phi - \phi_S)F_{LT}^{\cos(\phi - \phi_S)} \right]
\]

\[
+ \sqrt{2\epsilon(1 - \epsilon)} \cos(\phi_S)F_{LT}^{\cos(\phi_S)} + \sqrt{2\epsilon(1 - \epsilon)} \cos(2\phi - \phi_S)F_{LT}^{\cos(2\phi - \phi_S)} \right] \}
\]

\[ F_{LT}^{\cos(\phi_h - \phi_S)} = C \left[ \frac{\hat{h} \cdot \mathbf{p}_T}{M} g_{1T} D_1 \right] \]

Describes the probability to find longitudinally polarized quarks in a transversely polarized nucleon!

- requires interference between wave funct. components that differ by 1 unit of OAM
- Can be accessed in LT DSAs

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The $\cos(\phi-\phi_S)$ amplitudes

\[ \propto g_{1T}(x, p_T^2) \otimes D_1(z, k_T^2) \]

- slightly positive?
- consistent with zero
- positive!! similar observations from Hall-A and COMPASS
- slightly positive?
- consistent with zero
Selected higher-twist 1-hadron SIDIS results
Subleading twist

\[
\frac{d\sigma^h}{dx \, dy \, d\phi S \, dz \, d\phi \, dP_{h\perp}^2} = \alpha^2 \frac{y^2}{xyQ^2} \frac{2(1-\epsilon)}{2(1-\epsilon)} \left( 1 + \gamma^2 \right) = \frac{2\epsilon}{2M+h} \left[ F_{UU,T} + \epsilon F_{UU,L} \right. \\
+ \sqrt{2\epsilon (1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \\
+ \lambda_1 \left[ \sqrt{2\epsilon (1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\
+ S_L \left[ \sqrt{2\epsilon (1-\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\
+ S_L \lambda_1 \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon (1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\
+ S_T \left[ \sin(\phi - \phi S) \left( F_{UT,T}^{\sin(\phi - \phi S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi S)} \right) \right. \\
+ \epsilon \sin(\phi + \phi S) F_{UT}^{\sin(\phi + \phi S)} + \epsilon \sin(3\phi - \phi S) F_{UT}^{\sin(3\phi - \phi S)} \\
+ \sqrt{2\epsilon (1+\epsilon)} \sin(\phi S) F_{UT}^{\sin(\phi S)} \\
+ \sqrt{2\epsilon (1+\epsilon)} \sin(2\phi - \phi S) F_{UT}^{\sin(2\phi - \phi S)} \right] \\
+ S_T \lambda_1 \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi S) F_{LT}^{\cos(\phi - \phi S)} \right. \\
+ \sqrt{2\epsilon (1-\epsilon)} \cos(\phi S) F_{LT}^{\cos(\phi S)} \\
+ \sqrt{2\epsilon (1-\epsilon)} \cos(2\phi - \phi S) F_{LT}^{\cos(2\phi - \phi S)} \right]
\]

\[
F_{UT}^{\sin(\phi_S)} = \frac{2M}{Q} C \left\{ \left( x_f T D_1 - \frac{M_h h_1}{M} \frac{\tilde{H}}{z} \right) \right. \\
- \frac{k_T \cdot P_T}{2M M_h} \left[ \left( x h_T H_{1T} + \frac{M_h}{M} g_{1T} \right) \tilde{G}^{\perp}_{1T} \frac{1}{z} - \left( x h_T H_{1T} + \frac{M_h}{M} f_{1T} \right) \tilde{D}^{\perp}_{1T} \frac{1}{z} \right]\}
\]

Sensitive to worm-gear $\tilde{G}^{\perp}_{1T}$, sivers, transversity + higher-twist DF and FF

<table>
<thead>
<tr>
<th>quark</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>nucleon</td>
<td>$f_1$</td>
<td>$g_1$</td>
<td>$h_{1T}$</td>
</tr>
<tr>
<td>U</td>
<td></td>
<td>$\circ$</td>
<td>$-$</td>
</tr>
<tr>
<td>L</td>
<td></td>
<td>$\circ$</td>
<td>$\circ$</td>
</tr>
<tr>
<td>T</td>
<td>$f_{1T}^{\perp}$</td>
<td>$g_{1T}^{\perp}$</td>
<td>$h_{1T}$</td>
</tr>
</tbody>
</table>

Distribution Functions
Subleading-twist $\sin(\phi_S)$ Fourier component

- sensitive to worm-gear $g_{1T}$, Sivers function, Transversity, etc
- significant non-zero signal for $\pi^-$ and $K^-$!
Subleading-twist $\sin(\phi_S)$ Fourier component

- sensitive to worm-gear $g_{1T}$, Sivers function, Transversity, etc
- significant non-zero signal for $\pi^-$ and $K^-$!

Large and negative

Low-$Q^2$ amplitude larger

Hint of $Q^2$ dependence for $\pi^-$
$$F_{LU} \sin \phi$$

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi dP_{h_L}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ F_{UU,T} + \epsilon F_{UU,L} \right.$$  
$$+ \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right\}$$

$$\left. + \lambda_1 \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \right\}$$

$$+ S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_1 \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) \right.$$  
$$+ \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)}$$
$$+ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UT}^{\sin(\phi)}$$
$$+ \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right\}$$

$$+ S_T \lambda_1 \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \right.$$  
$$+ \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)}$$
$$+ \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right\}$$

Sensitive to $f_1$, Boer-Mulders + higher-twist DF and FF
\[ F_{LU} \sin \phi \]

\[
\frac{d\sigma^h}{dx \, dy \, d\phi_S \, dz \, d\phi \, dP_{h\perp}^2} = \frac{\alpha^2 y^2}{xyQ^2} 2 \frac{1 - \epsilon}{2x} \left( 1 + \frac{\gamma^2}{2x} \right)
\]

\[
\left\{ \begin{array}{l}
F_{UU,T} + \epsilon F_{UU,L} \\
\sqrt{2\epsilon (1 + \epsilon) \cos (\phi)} F_{UU}^{\cos (\phi)} + \epsilon \cos (2\phi) F_{UU}^{\cos (2\phi)} \\
\end{array} \right.
\]

+ \lambda_l \left[ \sqrt{2\epsilon (1 - \epsilon) \sin (\phi)} F_{LU}^{\sin (\phi)} \right]

+ S_L \left[ \sqrt{2\epsilon (1 + \epsilon) \sin (\phi)} F_{UL}^{\sin (\phi)} + \epsilon \sin (2\phi) F_{UL}^{\sin (2\phi)} \right]

+ S_L \lambda_l \left[ \sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2\epsilon (1 - \epsilon) \cos (\phi)} F_{LL}^{\cos (\phi)} \right]

+ S_T \left[ \sin (\phi - \phi_S) \left( F_{UT,T}^{\sin (\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin (\phi - \phi_S)} \right) + \epsilon \sin (\phi + \phi_S) F_{UT}^{\sin (\phi + \phi_S)} + \epsilon \sin (3\phi - \phi_S) F_{UT}^{\sin (3\phi - \phi_S)} + \sqrt{2\epsilon (1 + \epsilon) \sin (\phi_S)} F_{UT}^{\sin (\phi_S)} + \sqrt{2\epsilon (1 + \epsilon) \sin (2\phi - \phi_S)} F_{UT}^{\sin (2\phi - \phi_S)} \right]

+ S_T \lambda_l \left[ \sqrt{1 - \epsilon^2} \cos (\phi - \phi_S) F_{LT}^{\cos (\phi - \phi_S)} + \sqrt{2\epsilon (1 - \epsilon) \cos (\phi_S)} F_{LT}^{\cos (\phi_S)} + \sqrt{2\epsilon (1 - \epsilon) \cos (2\phi - \phi_S)} F_{LT}^{\cos (2\phi - \phi_S)} \right] \}


\[ \vec{e} p \rightarrow e \pi^+ X \]

5.5\% scale uncertainty

1996-2000 data

\[ \pi^+ \]

open circles 0.2<z<0.5

full circles 0.5<z<0.8

open squares: 0.8<z<1.0

\[ \pi^- \]

\[ \pi^0 \]
$F_{LU} \sin \phi$

\[
F_{LU}^{\sin \phi} = \frac{2M}{Q} \mathcal{C} \left[ -\frac{\hat{h} \cdot k_T}{M_h} \left( x e \frac{H^\perp}{z} + \frac{M_h}{M} \frac{\hat{G}^\perp}{z} \right) + \frac{\hat{h} \cdot p_T}{M} \left( x g D_1 + \frac{M_h}{M} \frac{h^\perp}{z} \frac{E}{z} \right) \right]
\]

$F_{LU}^{\sin \phi}$

H target, 2000-2007 data 0.2<z<0.7

Released yesterday!!
\[ F_{LU} \sin \phi \]

\[
F_{LU}^{\sin \phi_h} = \frac{2M}{Q} c \left[ -\frac{\hat{h} \cdot k_T}{M_h} \left( x e H_1^+ + \frac{M_h}{M} f_1 \frac{G^+}{z} \right) + \frac{\hat{h} \cdot p_T}{M} \left( x g^+ D_1 + \frac{M_h}{M} h_1^+ \frac{E}{z} \right) \right]
\]

D target, 2000-2007 data 0.2<z<0.7

Released yesterday!!
2-hadron SIDIS results

Following formalism developed by Steve Gliske

Find details in

Transverse Target Moments of Dihadron Production in Semi-inclusive Deep Inelastic Scattering at HERMES
S. Gliske, PhD thesis, University of Michigan, 2011
A short digression on di-hadron fragmentation functions

**Standard definition** of di-hadron FF assume no polarization of final state hadrons (pseudo-scalar mesons) or define mixtures of certain partial waves as new FFs.

In the **new formalism** there are only two di-hadron FFs. Names and symbols are entirely associated with the quark spin states ($D_1$ for $\chi = \chi'$ and $H_1^\perp$ (generalized Collins) for $\chi \neq \chi'$), whereas the partial waves of the produced hadrons ($|l_1 m_1\rangle, |l_2 m_2\rangle$) are associated with partial waves of FFs.

$$
D_1 = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos \vartheta) e^{im(\phi_R-\phi_k)} D_1^{\ell,m}(z, M_h, |k_T|)
$$

$$
H_1^\perp = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos \vartheta) e^{im(\phi_R-\phi_k)} H_1^\perp\ell,m(\tilde{z}, M_h, |k_T|)
$$

The cross-section is identical to the ones in literature, the only difference is the interpretation of the FFs:

$$
D_1^{[0,0]} = D_{1,OO} = \left(\frac{1}{4} D_{1,OO}^s + \frac{3}{4} D_{1,OO}^p\right)
$$

$$
D_1^{[1,0]} = D_{1,OL},
$$

$$
D_1^{[1,\pm1]} = D_{1,OT} + \frac{|k_T| |R|}{M_h^2} G_{1,OT},
$$

$$
D_1^{[2,0]} = \frac{1}{2} D_{1,LL},
$$

$$
D_1^{[2,\pm1]} = \frac{1}{2} \left(D_{1,LT} + \frac{|k_T| |R|}{M_h^2} G_{1,LT}\right),
$$

$$
D_1^{[2,\pm2]} = D_{1,TT} + \frac{1}{2} \frac{|k_T| |R|}{M_h^2} G_{1,TT},
$$

$$
H_1^{[0,0]} = H_{1,OO} = \frac{1}{4} H_{1,OO}^s + \frac{3}{4} H_{1,OO}^p,
$$

$$
H_1^{[1,1]} = H_{1,OT} + \frac{|R|}{|k_T|} H_{1,OT} = \frac{|R|}{|k_T|} H_{1,OT},
$$

$$
H_1^{[1,0]} = H_{1,OL},
$$

$$
H_1^{[1,-1]} = H_{1,OT},
$$

$$
H_1^{[2,2)} = H_{1,LT} + \frac{|R|}{|k_T|} H_{1,LT} = \frac{|R|}{|k_T|} H_{1,LT},
$$

$$
H_1^{[2,1)} = \frac{1}{2} H_{1,LT} + \frac{1}{2} \frac{|R|}{|k_T|} H_{1,LT} = \frac{1}{2} \frac{|R|}{|k_T|} H_{1,LT},
$$

$$
H_1^{[2,-2)} = H_{1,TT},
$$

$$
H_1^{[2,-1)} = H_{1,TT} + \frac{|R|}{|k_T|} H_{1,TT} = \frac{|R|}{|k_T|} H_{1,TT}.
$$
The di-hadron SIDIS cross-section

\[
d\sigma_{UT} = \frac{\alpha^2 M_h P_{h\perp}}{2\pi x y Q^2} \left(1 + \frac{\gamma^2}{2x}\right) |S_\perp|
\]

\[
\times \sum_{\ell=0}^2 \sum_{m=-\ell}^{\ell} \left\{ A(x, y) \left[ P_{\ell,m} \sin((m+1)\phi_h - m\phi_R - \phi_S) \right]
\right.
\]

\[
\times \left( F_{UT,T}^{P_{\ell,m}\sin((m+1)\phi_h - m\phi_R - \phi_S)} + \epsilon F_{UT,L}^{P_{\ell,m}\sin((m+1)\phi_h - m\phi_R - \phi_S)} \right)
\]

\[
+ B(x, y) \left[ P_{\ell,m} \sin((1-m)\phi_h + m\phi_R + \phi_S) F_{UT}^{P_{\ell,m}\sin((1-m)\phi_h + m\phi_R + \phi_S)} \right]
\]

\[
+ P_{\ell,m} \sin((3-m)\phi_h + m\phi_R - \phi_S) F_{UT}^{P_{\ell,m}\sin((3-m)\phi_h + m\phi_R - \phi_S)} \right]
\]

\[
+ V(x, y) \left[ P_{\ell,m} \sin(-m\phi_h + m\phi_R + \phi_S) F_{UT}^{P_{\ell,m}\sin(-m\phi_h + m\phi_R + \phi_S)} \right]
\]

\[
+ P_{\ell,m} \sin((2-m)\phi_h + m\phi_R - \phi_S) F_{UT}^{P_{\ell,m}\sin((2-m)\phi_h + m\phi_R - \phi_S)} \right\}.
\]

\( l \) and \( m \) correspond to the \( l \) and \( m \) in \(|l m\rangle\) angular momentum state of the hadron

Considering all terms \( d\sigma_{UU}, d\sigma_{LU}, d\sigma_{UL}, d\sigma_{LL}, d\sigma_{UT}, d\sigma_{LT} \) there are 144 non-zero structure functions at twist-3 level. The most known is

\[
F_{UT}^{P_{\ell,m}\sin((1-m)\phi_h + m\phi_R + \phi_S)} = -i \left[ \frac{|k_T|}{M_h} \cos ((m - 1)\phi_h - \phi_p - m\phi_k) \ h_1 H_{1,\ell,m}^{(1)} \right]
\]

which for \( l = 1 \) and \( m = 1 \) reduces to the well known colliner \( F_{UT}\sin \theta \sin(\phi_R + \phi_S) \) related to transversity.
The di-hadron SIDIS cross-section

Published $\pi^+\pi^-$ Results

- independent way to access transversity
- significantly positive amplitudes
- $1^{st}$ evidence of non zero dihadron FF
- no convolution integral involved
- limited statistical power (v.r.t. 1 hadron)

$$\sigma_{UT} \propto S_T \sin\theta \sin(\phi_{RL} + \phi_S) \sum_q e_q^2 \delta q \mathcal{H}_{i,q}$$
The di-hadron SIDIS cross-section

Published $\pi^+\pi^-$ Results

- independent way to access transversity
- significantly positive amplitudes
- $1^{st}$ evidence of non zero dihadron FF
- no convolution integral involved
- limited statistical power (v.r.t. 1 hadron)
- signs are consistent for all $\pi\pi$ species
- statistics much more limited for $\pi^+\pi^0$
- despite uncertainties may still help to constrain global fits and may assist in $u-d$ flavor separation

New $\pi^\pm\pi^0$ Results

- New tracking, new PID, use of $\phi_R$ rather than $\phi_{R\perp}$
- Different fitting procedure and function
- Acceptance correction
A short digression on the Lund/Artru string fragmentation model
(a phenomenological explanation of the Collins effect)

In the cross-section the Collins FF is always paired with a distribution function involving a transverse polarized quark.

1. Assume u quark and proton have (transverse) spin aligned in the direction \( \phi_S = \pi/2 \). The model assumes that the struck quark is initially connected with the remnant via a gluon-flux tube (string).

2. When the string breaks, a \( q \bar{q} \) pair is created with vacuum quantum numbers \( J^P = 0^+ \). The positive parity requires that the spins of \( q \) and \( \bar{q} \) are aligned, thus an OAM \( L = 1 \) has to compensate the spins.

3. This OAM generates a transverse momentum of the produced pseudo-scalar meson (e.g. \( \pi^+ \)) and deflects the meson to the left side w.r.t. the struck quark direction, generating left-right azimuthal asymmetries.
A short digression on the Lund/Artru string fragmentation model

Relative to the proton transv. spin, the fragmenting quark can have spin parallel or antiparallel to $|\frac{1}{2}, \pm \frac{1}{2}\rangle$

Then combining the spins of the formed di-quark systems one can get:

$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0 \Rightarrow \begin{cases} 
1 \text{ spin 0 state } |0, 0\rangle & 1 \text{ pseudo-scalar meson (PSM)} \\
3 \text{ spin 1 states } \begin{cases} 
|1, 0\rangle & 1 \text{ Longitudinal VM} \\
|1, \pm 1\rangle & 2 \text{ transverse VM}
\end{cases}
\end{cases}$$

**Lund/Artru prediction at the amplitude level:** the asymmetry for PSM has opposite sign to that for transversely polarized VM (left vs. right side), and the amplitude for $|1, 0\rangle$ is 0

Lund/Artru model makes predictions for the individual di-hadrons, but the Collins function includes pairs of di-hadrons

→ to make predictions for the Collins function one needs to consider the cross-section level, i.e. the sum of contributing amplitudes times their complex conjugate

Using the Clebsch-Gordan algebra one obtains: $|1, \pm 1\rangle|1, \pm 1\rangle \equiv |2, \pm 2\rangle$

**Lund/Artru prediction at the cross-section level:** the $|2, \pm 2\rangle$ partial waves of the Collins func. for SIDIS VM production have the opposite sign as the respective PS Collins func.
“gluon radiation model” vs. Lund/Artru model

The Lund/Artru model only accounts for favored Collins fragmentation. An extension of the model (the **gluon radiation model**), elaborated by S. Gliske accounts for the disfavored case

1. Struck quark emits a gluon in such a way that most of its momentum is transferred to the gluon
2. The struck quark then becomes part of the remnant
3. The radiated gluon produces a $q\bar{q}$ pair that eventually converts into a meson
4. For PSM the di-quark must interact further with the remnant to get the PSM quantum numbers. In case of VM the di-quark directly forms the meson

![Lund/Artu](image1)

- Di-quark has q.n. of vacuum
- **Struck quark** joins the anti-quark in the final state → **favored fragment**.

**Prediction:** the $|2, \pm 2\rangle$ partial wave of the Collins funct. for SIDIS VM production have the opposite sign as the respective PS Collins function

![Gluon radiation](image2)

- Di-quark has q.n. of observed final state
- **Produced quark** joins the anti-quark in the final state → **disfavored fragment**.

**Prediction:** the disfavored $|2, \pm 2\rangle$ Collins frag. also is expected to have opposite sign as the respective PS Collins function.

\[
\text{Models predict: fav = disfav for VM}
\]

\[
\text{Data say: fav } \approx - \text{ disfav for PSM (Collins } \pi^+ \text{ vs. } \pi^- )
\]
...and now let’s look at the results

<table>
<thead>
<tr>
<th>Fragment. process</th>
<th>Fav/disfav</th>
<th>Deflection</th>
<th>Sign of amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u \to \pi^+$</td>
<td>fav PSM</td>
<td>left ($\phi_h \to 0$)</td>
<td>$&gt; 0$ (Collins $\pi^+$)</td>
</tr>
<tr>
<td>$u \to \pi^-$</td>
<td>disfav PSM</td>
<td>right ($\phi_h \to \pi$)</td>
<td>$&lt; 0$ (Collins $\pi^-$)</td>
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<tr>
<td>$u \to \rho^+ \to \pi^+\pi^0$</td>
<td>fav VM</td>
<td>right ($\phi_h \to \pi$)</td>
<td>$&lt; 0$</td>
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<td>right ($\phi_h \to \pi$)</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$u \to \rho^0 \to \pi^+\pi^-$</td>
<td>mixed VM</td>
<td>right ($\phi_h \to \pi$)</td>
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From data

From models
...and now let’s look at the results

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<td>$u \rightarrow \rho^+ \rightarrow \pi^+\pi^0$</td>
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<td>right ($\phi_h \rightarrow \pi$)</td>
<td>0 or $&lt; 0$</td>
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</table>

$|2, -2\rangle$ consistent with zero for all flavors
Not in contraddiction with models: if the transversity function causes the fragmenting quark to have positive polarization than Collins $|2, -2\rangle$ must be zero as this partial wave requires fragmenting quark with negative polarization

$|2, +2\rangle$ consistent with model expect:
- No signal outside $\rho$-mass bin
  $\rightarrow$ no non-resonant pion-pairs in p-wave
- Negative for $\rho^\pm$ (model predictions)
- very small for $\rho^0$ (consistent with small Collins $\pi^0$)
Back-up
The $\cos \phi$ amplitudes

A. Airapetian et al, Phys. Rev. D 87 (2013) 012010

Analysis multi-dimensional in x, y, z, and Pt
Create your own projections of results through: http://www-hermes.desy.de/cosnphi/
\[ F_{UL}^\sin \phi \]

\[
\frac{d\sigma^h}{dx \, dy \, d\phi_S \, dz \, d\phi \, dP_{h\perp}^2} = \frac{\alpha^2}{x y Q^2} \frac{y^2}{2 (1 - \epsilon)} \left( 1 + \frac{\gamma^2}{2 x} \right) \left\{ \begin{array}{l}
F_{UU,T} + \epsilon F_{UU,L} \\
+ \sqrt{2 \epsilon (1 + \epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \\
+ \lambda_l \left[ \sqrt{2 \epsilon (1 - \epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]
\end{array} \right.
\]

\[
+ S_L \left[ \sqrt{2 \epsilon (1 + \epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]
\]

\[
+ S_L \lambda_l \left[ \sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2 \epsilon (1 - \epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]
\]

\[
+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) \\
+ \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \\
+ \sqrt{2 \epsilon (1 + \epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\
+ \sqrt{2 \epsilon (1 + \epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right]
\]

\[
+ S_T \lambda_l \left[ \sqrt{1 - \epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \right.
\]

\[
+ \sqrt{2 \epsilon (1 - \epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\
+ \sqrt{2 \epsilon (1 - \epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right].
\]

Worm-gear $h_{1L}^\perp$

\[
\frac{d\sigma^h}{dx \, dy \, d\phi \, dz \, dP_{h,\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{\gamma^2}{2(1 - \epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ \begin{array}{l}
F_{UU,T} + \epsilon F_{UU,L} \\
+ \sqrt{2\epsilon(1 + \epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} 
\end{array} \right.
\]

\[+ \lambda_I \left[ \sqrt{2\epsilon(1 - \epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \]

\[+ S_L \left[ \sqrt{2\epsilon(1 + \epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \]

\[+ S_L \lambda_I \left[ \sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2\epsilon(1 - \epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \]

\[+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} + \sqrt{2\epsilon(1 + \epsilon)} \sin(\phi S) F_{UT}^{\sin(\phi S)} + \sqrt{2\epsilon(1 + \epsilon)} \sin(2\phi - \phi S) F_{UT}^{\sin(2\phi - \phi S)} \right] \]

\[+ S_T \lambda_I \left[ \sqrt{1 - \epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} + \sqrt{2\epsilon(1 - \epsilon)} \cos(\phi S) F_{LT}^{\cos(\phi S)} + \sqrt{2\epsilon(1 - \epsilon)} \cos(2\phi - \phi S) F_{LT}^{\cos(2\phi - \phi S)} \right] \}

\[
F_{UL}^{\sin 2\phi_h} = C \left[ -\frac{2(\hat{h} \cdot k_T)(\hat{h} \cdot p_T) - k_T \cdot p_T}{M M_h} h_{1L}^\perp H_{1T}^\perp \right]
\]

Describes the probability to find transversely polarized quarks in a longitudinally polarized nucleon
The sin(2\(\phi\)) amplitude \(\propto h_{1L}^\perp(x, p_T^2) \otimes H_1^\perp(z, k_T^2)\)

Deuterium target

Amplitudes consistent with zero for all mesons and for both H and D targets


Pretzelosity

\[ \frac{d\sigma^h}{dx \, dy \, d\phi \, dP_{h \perp}^2} = \frac{e^2}{x y Q^2} \left( 1 + \frac{\gamma^2}{2x} \right) \]

\[ \left\{ \begin{array}{l}
F_{UU,T} + \epsilon F_{UU,L} \\
+ \sqrt{2\epsilon (1 + \epsilon)} \cos (\phi) F_{UU}^{\cos (\phi)} + \epsilon \cos (2\phi) F_{UU}^{\cos (2\phi)} \\
+ \lambda_l \left[ \sqrt{2\epsilon (1 - \epsilon)} \sin (\phi) F_{LU}^{\sin (\phi)} \right] \\
+ S_L \left[ \sqrt{2\epsilon (1 - \epsilon)} \sin (\phi) F_{UL}^{\sin (\phi)} + \epsilon \sin (2\phi) F_{UL}^{\sin (2\phi)} \right] \\
+ S_L \lambda_l \left[ \sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2\epsilon (1 - \epsilon)} \cos (\phi) F_{LL}^{\cos (\phi)} \right] \\
+ S_T \left[ \sin (\phi - \phi S) \left( F_{UT,T}^{\sin (\phi - \phi S)} + \epsilon F_{UT,L}^{\sin (\phi - \phi S)} \right) \right. \\
+ \left. \epsilon \sin (\phi + \phi S) F_{UT}^{\sin (\phi + \phi S)} + \epsilon \sin (3\phi - \phi S) F_{UT}^{\sin (3\phi - \phi S)} \right] \\
+ \left. \sqrt{2\epsilon (1 + \epsilon)} \sin (\phi S) F_{UT}^{\sin (\phi S)} \right. \\
+ \left. \sqrt{2\epsilon (1 + \epsilon)} \sin (2\phi - \phi S) F_{UT}^{\sin (2\phi - \phi S)} \right] \\
+ S_T \lambda_l \left[ \sqrt{1 - \epsilon^2} \cos (\phi - \phi S) F_{LT}^{\cos (\phi - \phi S)} \right. \\
+ \left. \sqrt{2\epsilon (1 - \epsilon)} \cos (\phi S) F_{LT}^{\cos (\phi S)} \right. \\
+ \left. \sqrt{2\epsilon (1 - \epsilon)} \cos (2\phi - \phi S) F_{LT}^{\cos (2\phi - \phi S)} \right] \end{array} \right\} \]

Describes correlation between quark transverse momentum and transverse spin in a transversely pol. nucleon.

- Sensitive to **non-spherical shape** of the nucleon.

### Distribution Functions

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<tr>
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<th>U</th>
<th>L</th>
<th>T</th>
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</thead>
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<td><strong>U</strong></td>
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<td>$g_1$</td>
<td>$h_{1L}$</td>
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<td><strong>L</strong></td>
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<td><strong>T</strong></td>
<td>$h_{1}$</td>
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### Fragmentation Functions

<table>
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<tr>
<td><strong>h</strong></td>
<td>$D_1$</td>
<td>$H_{1L}$</td>
<td>$H_{1}$</td>
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</table>
The sin(3φ-φₕ) amplitude $\propto h_{1T}^{\perp}(x, p_T^2) \otimes H_1^{\perp}(z, k_T^2)$

All amplitudes consistent with zero

...suppressed by two powers of $P_{h,\perp}$ w.r.t. Collins and Sivers amplitudes
The only TMD that is both chiral-even and naïve-T-even requires interference between wave funct. components that differ by 1 unit of OAM

Many models support simple relations among $g_{1T}^q$ and other TMDs:

1. $g_{1T}^q = -h_{1L}^{-1}$ (also supported by Lattice QCD and first data)

2. $g_{1T}^{q(1)}(x) \approx \frac{1}{x} \int_{x}^{1} \frac{dy}{y} g_{1}^{q}(y)$ (Wandzura-Wilczek appr.)
Probing $g_{1T}$ through Double Spin Asymmetries

$$F_{LT}^{\cos(\phi_h-\phi_S)} = C \left[ \frac{\hat{h} \cdot p_T}{M} g_{1T} D_1 \right]$$

$$F_{LT}^{\cos \phi_S} = \frac{2M}{Q} C \left\{ - \left( x g_T D_1 + \frac{M_h}{M} h_1 \tilde{E} \right) 
+ \frac{k_T \cdot p_T}{2MM_h} \left[ \left( x e_T H_1^\perp - \frac{M_h}{M} g_{1T} D_1^\perp \right) + \left( x e_T H_1^\perp + \frac{M_h}{M} f_{1T} \tilde{G}^\perp \right) \right] \right\}$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = \frac{2M}{Q} C \left\{ - \frac{2 (\hat{h} \cdot p_T)^2 - p_T^2}{2M^2} \left( x g_T D_1 + \frac{M_h}{M} h_1 \tilde{E} \right) 
+ \frac{2 (\hat{h} \cdot k_T) (\hat{h} \cdot p_T) - k_T \cdot p_T}{2MM_h} \left[ \left( x e_T H_1^\perp - \frac{M_h}{M} g_{1T} D_1^\perp \right) 
- \left( x e_T H_1^\perp + \frac{M_h}{M} f_{1T} \tilde{G}^\perp \right) \right] \right\}$$

The simplest way to probe worm-gear $g_{1T}$ is through the $\cos(\phi - \phi_S)$ Fourier component
The $\cos(\phi_S)$ Fourier component

\[ 2 \langle \cos(\phi_S) \rangle^\pi_{L} \]

- $\pi^+$: \(\approx 0\)
- $\pi^0$: \(\approx 0\)
- $\pi^-$: \(\approx 0\)

\[ 2 \langle \cos(\phi_S) \rangle^K_{L} \]

- $K^+$: \(\approx 0\)
- $K^-$: \(\approx 0\)

Sign change?
The $\cos(2\phi-\phi_S)$ Fourier component

For each of the following particles, the 2-fold angular correlation $2\langle \cos(2\phi-\phi_S) \rangle$ is plotted against various variables such as $x$, $z$, and $P_{h\perp}$ (in GeV). The data is from the HERMES experiment, with preliminary uncertainty scaling. The arrows indicate that the correlation is approximately zero for $\pi^+$, $\pi^0$, $\pi^-$, $K^+$, and $K^-$.
The \( \sin(2\phi + \phi_S) \) Fourier component

- arises solely from longitudinal (w.r.t. virtual photon direction) component of the target spin

- related to \( \langle \sin(2\phi) \rangle_{UL} \) Fourier comp:
  \[ 2\langle \sin(2\phi + \phi_S) \rangle_{LT} \propto \frac{1}{2} \sin(\mathcal{G}_{LT}^*) \langle \sin(2\phi) \rangle_{UL} \]

- sensitive to **worm-gear** \( h_{1L} \)

- suppressed by one power of \( P_{h_{1L}} \) w.r.t. Collins and Sivers amplitudes

- no significant signal observed (except maybe for \( K^+ \))
The subleading-twist $\sin(2\phi - \phi_S)$ Fourier component

- sensitive to worm-gear $\mathcal{g}_{1T}$, Pretzelosity and Sivers function:

\[
\propto W_1(p_T, k_T, P_{h\perp}) \left( x f_T^D D_1 - \frac{M}{M} h_{1T}^\perp \frac{\tilde{H}}{z} \right) \\
- W_2(p_T, k_T, P_{h\perp}) \left[ \left( x h_T H_1^\perp + \frac{M}{M} g_{1T} \frac{\tilde{G}^\perp}{z} \right) \\
+ \left( x h_T H_1^\perp - \frac{M}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z} \right) \right]
\]

- suppressed by one power of $P_{h\perp}$ w.r.t. Collins and Sivers amplitudes

- no significant non-zero signal observed
The HERMES experiment at HERA

hadron separation

TRD, Calorimeter, preshower, RICH: lepton-hadron > 98%

Aerogel $n=1.03$

$\pi \sim 98\%, K \sim 88\%, P \sim 85\%$

$C_4F_{10} \ n=1.0014$
Siver amplitudes: additional studies

No systematic shifts observed between high and low Q² amplitudes for both π⁺ and K⁺

No indication of important contributions from exclusive VM
The pion-difference asymmetry

Contribution by decay of exclusively produced vector mesons ($\rho^0, \omega, \phi$) is not negligible (6-7% for pions and 2-3% for kaons), though substantially limited by the requirement $z<0.7$.

A new observable

$$A_{UT}^{\pi^+ - \pi^-} (\phi, \phi_S) \equiv \frac{1}{P_T} \left( \sigma_{U\uparrow}^{\pi^+} - \sigma_{U\uparrow}^{\pi^-} \right) - \left( \sigma_{U\downarrow}^{\pi^+} - \sigma_{U\downarrow}^{\pi^-} \right)$$

Contribution from exclusive $\rho^0$ largely cancels out!

- significantly positive Sivers and Collins amplitudes are obtained
- measured amplitudes are not generated by exclusive VM contribution