Recent results on TMDs from the HERMES Experiment

Luciano L. Pappalardo

University of Ferrara
The nucleon tomography

**TMDs** \( f(x, p_T) \)  
**GPDs** \( H(x, \xi, t) \)

### 3D picture in momentum space

#### Semi-inclusive DIS

- Mother Wigner function:
  - Describes full phase-space distributions of partons, but not accessible experimentally

### 3D picture in coordinate space

#### Exclusive reactions

\[
\int d^3r \quad W(x, p_T, r) \quad \int d^2p_T
\]

A.B., F. Conti, M. Radici, PRD78 (08)

QCDSF/UKQCD, PRL 98 (07)
The nucleon tomography

TMDs

\[ f(x, p_T) \]

3D picture in momentum space

- Depend on \( x \) and \( p_T \)
- Describe correlations between \( p_T \) and quark or nucleon spin (spin-orbit correlations)

<table>
<thead>
<tr>
<th>quark</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>nucleon</td>
<td>( f_1 )</td>
<td>( g_1 )</td>
<td>( h_1 )</td>
</tr>
</tbody>
</table>

Semi-inclusive DIS

Diagonal elements survive integration over \( p_T \)

- Momentum and Helicity well known from inclusive DIS
- Transversity accessed only recently in SIDIS, still poorly known (differs from helicity due to relativistic effects)

A.B., F. Conti, M. Radici, PRD78 (08)
The nucleon tomography

TMDs $f(x, p_T)$

3D picture in momentum space

- Momentum and Helicity well known from inclusive DIS
- Now focus on $p_T$ dependence
- Transversity accessed only recently in SIDIS, still poorly known (differs from helicity due to relativistic effects)
- Sivers and BM: T-odd $\rightarrow$ require non-trivial (process-dependent!) gauge-link. Recently probed in SIDIS. Non zero and strongly flavour dependent
- $w-g g_{1T}$: hint of non-zero signal. Very preliminary access.
- $w-g h_{1L}$: zero at HERMES and COMPASS, significant amplitudes at CLAS!
- pretzelosity consistent with zero (HERMES, COMPASS)

A.B., F. Conti, M. Radici, PRD78 (08)

- Depend on $x$ and $p_T$
- Describe correlations between $p_T$ and quark or nucleon spin (spin-orbit correlations)

Boer Mulders
Sivers
worm-gears
pretzelosity
Accessing the TMDs

**TMDs** \( f(x, p_T) \)

3D picture in momentum space

- Depend on \( x \) and \( p_T \)

- Describe correlations between \( p_T \) and quark or nucleon spin (spin-orbit correlations)

A.B., F. Conti, M. Radici, PRD78 (08)

**TMDs**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( p_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>4.4</td>
</tr>
</tbody>
</table>

TMD factorization for \( P_{h_1} \ll Q \)

\[
\mathcal{O}^{ep\to ehX} = \sum_q \mathcal{O}^{eq\to eq} \mathcal{O}^{DF} \mathcal{O}^{FF}
\]

**Fragmentation Functions**

<table>
<thead>
<tr>
<th>( h )</th>
<th>( x )</th>
<th>( p_T )</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>U</td>
<td>( D_j )</td>
</tr>
<tr>
<td>T</td>
<td>( H_j^\perp )</td>
<td></td>
</tr>
</tbody>
</table>
The SIDIS cross-section

\[ \frac{d\sigma^h}{dx dy d\phi_S dz d\phi dP^2_{h\perp}} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \]

\[
\begin{align*}
\{ & F_{UU,T} + \epsilon F_{UU,L} \\
& + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \\
& + \lambda_1 \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\
& + S_L \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{UL}^{\cos(\phi)} \right] \\
& + S_T \lambda_1 \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \right] \\
& \quad + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \\
& \quad + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\
& \quad + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \}
\end{align*}
\]
The SIDIS cross-section

\[
\frac{d^2 \sigma}{dx \, dy \, d\phi \, dz \, dP^2_{h1}} = \frac{\alpha^2}{x y Q^2} \frac{y^2}{2 (1 - \varepsilon)} \left( 1 + \frac{\gamma^2}{2x} \right)
\]

\[
\left\{ \begin{array}{l}
F_{UU,T} + \varepsilon F_{UU,L} \\
+ \sqrt{2 \varepsilon (1 + \varepsilon)} \cos(\phi) F_{UU}^{\cos}(\phi) + \varepsilon \cos(2\phi) F_{UU}^{\cos}(2\phi) \\
+ \lambda_l \left[ \sqrt{2 \varepsilon (1 - \varepsilon)} \sin(\phi) F_{LU}^{\sin}(\phi) \right] \\
+ S_L \left[ \sqrt{2 \varepsilon (1 + \varepsilon)} \sin(\phi) F_{UL}^{\sin}(\phi) + \varepsilon \sin(2\phi) F_{UL}^{\sin}(2\phi) \right] \\
+ S_L \lambda_l \left[ \sqrt{1 - \varepsilon^2} F_{LL} + \sqrt{2 \varepsilon (1 - \varepsilon)} \cos(\phi) F_{LU}^{\cos}(\phi) \right] \\
+ S_T \left[ \sin(\phi - \phi s) \left( F_{UT,T}^{\sin}(\phi - \phi s) + \varepsilon F_{UT,L}^{\sin}(\phi - \phi s) \right) \\
+ \varepsilon \sin(\phi + \phi s) F_{UT}^{\sin}(\phi + \phi s) + \varepsilon \sin(3\phi - \phi s) F_{UT}^{\sin}(3\phi - \phi s) \\
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+ S_T \lambda_l \left[ \sqrt{1 - \varepsilon^2} \cos(\phi - \phi s) F_{LT}^{\cos}(\phi - \phi s) \\
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The SIDIS cross-section

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+ S_L \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \varepsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\
+ S_L \lambda_L \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\
+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) \\
+ \varepsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \varepsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right) \\
+ \sqrt{2\varepsilon(1+\varepsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \right. \\
\left. + \sqrt{2\varepsilon(1-\varepsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right] \\
+ S_T \lambda_L \left[ \sqrt{1-\varepsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \right. \\
\left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \right) \right\} \\
+ \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right\}
\]

Leading twist
Sub-leading Twist
Selected results (1)

The Naive-T-odd TMDs

<table>
<thead>
<tr>
<th>TMDs</th>
<th>quark</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
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<tbody>
<tr>
<td>nucleon</td>
<td>U</td>
<td>$f_1$</td>
<td></td>
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<td></td>
<td>L</td>
<td>$g_1$</td>
<td>$h_{1L}^+$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>$f_{1T}^+$</td>
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Sivers

Boer Mulders
Boer-Mulders function $h_1^\perp$

\[
\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d^2 p_T^h} = \frac{\alpha^2}{xy Q^2} \frac{y^2}{2(1-\varepsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ \begin{array}{l}
F_{UU,T} + \epsilon F_{UU,L} \\
+ \sqrt{2\epsilon (1+\epsilon)} \cos(\phi) F_{UU}^{\cos}(\phi) + \epsilon \cos(2\phi) F_{UU}^{\cos}(2\phi) \\
+ \lambda_L \left[ \sqrt{2\epsilon (1-\epsilon)} \sin(\phi) F_{LU}^{\sin}(\phi) \right] \\
+ S_L \left[ \sqrt{2\epsilon (1+\epsilon)} \sin(\phi) F_{UL}^{\sin}(\phi) + \epsilon \sin(2\phi) F_{UL}^{\sin}(2\phi) \right] \\
+ S_L \lambda_L \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon (1-\epsilon)} \cos(\phi) F_{LL}^{\cos}(\phi) \right] \\
+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin}(\phi - \phi_S) + \epsilon F_{UT,L}^{\sin}(\phi - \phi_S) \right) \\
+ \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin}(\phi + \phi_S) + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin}(3\phi - \phi_S) \\
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+ S_T \lambda_L \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos}(\phi - \phi_S) \\
+ \sqrt{2\epsilon (1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos}(\phi_S) \\
+ \sqrt{2\epsilon (1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos}(2\phi - \phi_S) \right] \end{array} \right\}
\]

Naive-T-odd & Chiral-odd
Describes correlation between quark transverse momentum and transverse spin in unpolarized nucleon

\[ \propto h_1^\perp \otimes H_1^\perp \]

B-M effect [PRD 57 (1998)]
Boer-Mulders function $h_1^\perp$

$$\frac{d\sigma^h}{dxdydzd\phi dP_{h1}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \varepsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right\}$$

$$+ \lambda I \left[ \sqrt{2\varepsilon(1-\varepsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[ \sqrt{2\varepsilon(1-\varepsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \varepsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda I \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$+ S_T \left[ \sin(\phi - \phi S) \left( F_{UT,T}^{\sin(\phi - \phi S)} + \varepsilon F_{UT,L}^{\sin(\phi - \phi S)} \right) + \varepsilon \sin(2\phi - \phi S) F_{UT}^{\sin(2\phi - \phi S)} \right]$$

$$+ S_T \lambda I \left[ \sqrt{1-\varepsilon^2} \cos(\phi - \phi S) F_{LT}^{\cos(\phi - \phi S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(\phi S) F_{LT}^{\cos(\phi S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi - \phi S) F_{LT}^{\cos(2\phi - \phi S)} \right] \right\}$$

Naive-T-odd & Chiral-odd
Describes correlation between quark transverse momentum and transverse spin in unpolarized nucleon

$$\propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} [f_1 \otimes D_1 + \ldots]$$

B-M effect
[PRD 57 (1998)]

Cahn effect
[PLB 78 (1978)]
Boer-Mulders function $h_1^\perp$

$$\frac{d\sigma^h}{dx \; dy \; d\phi \; dz \; dP_{h_1}^2} = \frac{\alpha^2}{x y Q^2} \frac{y^2}{2 (1 - \varepsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon (1 + \varepsilon)} \cos (\phi) F_{UU}^{\cos (\phi)} + \varepsilon \cos (2\phi) F_{UU}^{\cos (2\phi)} \right\}$$

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$$\left. + \sqrt{2\varepsilon (1 + \varepsilon)} \sin (\phi S) F_{UT}^{\sin (\phi S)} \right) \left. + \sqrt{2\varepsilon (1 + \varepsilon)} \sin (2\phi - \phi S) F_{UT}^{\sin (2\phi - \phi S)} \right\}$$

$$+ S_T \lambda_1 \left[ \sqrt{1 - \varepsilon^2} \cos (\phi - \phi S) F_{LT}^{\cos (\phi - \phi S)} \right. \right.$$  

$$\left. + \sqrt{2\varepsilon (1 - \varepsilon)} \cos (\phi S) F_{LT}^{\cos (\phi S)} \right) \left. + \sqrt{2\varepsilon (1 - \varepsilon)} \cos (2\phi - \phi S) F_{LT}^{\cos (2\phi - \phi S)} \right\}$$

Naive-T-odd & Chiral-odd  
Describes correlation between quark transverse momentum and transverse spin in unpolarized nucleon

$$\propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} \left[ f_1 \otimes D_1 + \cdots \right]$$

**B-M effect**  
[PRD 57 (1998)]

**Cahn effect**  
[PLB 78 (1978)]

Interaction dependent terms
The cos2\(\phi\) amplitudes

\[
\propto h_1^+ \otimes H_1^+ + \frac{1}{Q^2} [f_1 \otimes D_1 + \ldots]
\]

- Amplitudes are significant → clear evidence of BM effect
- Similar results for H & D indicate \(h_1^{+,u} \approx h_1^{+,d}\)
- Opposite sign for \(\pi^+ / \pi^-\) consistent with opposite signs of fav/unfav Collins

A. Airapetian et al, Phys. Rev. D 87 (2013) 012010
The cos2\(\phi\) amplitudes

A. Airapetian et al, Phys. Rev. D 87 (2013) 012010

\[ \propto h_1^+ \otimes H_1^+ + \frac{1}{Q^2} [f_1 \otimes D_1 + \ldots] \]

- Amplitudes are significant → clear evidence of BM effect
- similar results for H & D indicate \( h_1^{+,u} \approx h_1^{+,d} \)
- Opposite sign for \( \pi^+ / \pi^- \) consistent with opposite signs of fav/unfav Collins

- \( K^+ / K^- \) amplitudes are larger than for pions, have different kinematic dependencies than pions and have same sign
- different role of Collins FF for pions and kaons?
- Significant contribution from scattering off strange quarks?

Analysis multi-dimensional in x, y, z, and Pt
Create your own projections of results through: http://www-hermes.desy.de/cosnphi/
The \( \cos \phi \) amplitudes

\[ \alpha + \frac{1}{Q} [h_1^+ \otimes H_1^+ + f_1 \otimes D_1 \ldots] \]

- Significant and of same sign (Chan effect expected to be weekly flavor dependent)
- Clear rise with \( z \) for \( \pi^+ \) & \( \pi^- \) and \( P_{h\perp} \) for \( \pi^+ \) (Chan)
- Different \( P_{h\perp} \) dependence of \( \pi^+ \) & \( \pi^- \) indicates contributions of flavor dependent effects (e.g. BM) for \( \pi^- \)
- \( K^+ \) amplitudes larger than \( \pi^+ \)
- \( K^- \approx 0 \) different than \( K^+ \) (in contrast to \( \cos 2\phi \))
- Significant contrib from interaction dependent terms?

Analysis multi-dimensional in \( x, y, z, \)and \( Pt \)

Create your own projections of results through: \( \text{http://www-hermes.desy.de/cosnphi/} \)
Sivers function $f_{1T}^{\perp}$

\[
\frac{d\sigma^h}{dx\,dy\,d\phi_S\,dz\,d\phi\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ \begin{array}{l}
F_{UU, T} + \varepsilon F_{UU, L} + 2\varepsilon (1+\varepsilon) \cos(\phi) F_{UU}^{\cos(\phi)} + \varepsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \\
+ \lambda_l \left[ \sqrt{2\varepsilon (1-\varepsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\
+ S_L \left[ \sqrt{2\varepsilon (1+\varepsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \varepsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\
+ S_L \lambda_l \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon (1-\varepsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\
+ S_T \left[ \sin(\phi - \phi S) \left( F_{UT, T}^{\sin(\phi - \phi S)} + \varepsilon F_{UT, L}^{\sin(\phi - \phi S)} \right) + \varepsilon \sin(\phi + \phi S) F_{UT}^{\sin(\phi + \phi S)} + \varepsilon \sin(3\phi - \phi S) F_{UT}^{\sin(3\phi - \phi S)} + \sqrt{2\varepsilon (1+\varepsilon)} \sin(\phi S) F_{UT}^{\sin(\phi S)} + \sqrt{2\varepsilon (1+\varepsilon)} \sin(2\phi - \phi S) F_{UT}^{\sin(2\phi - \phi S)} \right] \\
+ S_T \lambda_l \left[ \sqrt{1-\varepsilon^2} \cos(\phi - \phi S) F_{LT}^{\cos(\phi - \phi S)} + \sqrt{2\varepsilon (1-\varepsilon)} \cos(\phi S) F_{LT}^{\cos(\phi S)} + \sqrt{2\varepsilon (1-\varepsilon)} \cos(2\phi - \phi S) F_{LT}^{\cos(2\phi - \phi S)} \right] \end{array} \right\}
\]

Describes correlation between quark transverse momentum and nucleon transverse polarization

$\propto f_{1T}^{\perp} \otimes D_1$

[Sivers effect [PRD 41 (1990)]]
Sivers amplitudes

\[ \propto f_{17}^\perp \otimes D_1 \]

- **Significantly positive**
- **Slightly positive** (isospin-symmetry)
- **Consistent with zero**

Consistent with Sivers function of opposite sign for u and d quarks


[Anselmino et al., EPJA 3, 2009]
Sivers amplitudes

significant positive
slightly positive (isospin-symmetry)
consistent with zero
Larger than $\pi^+$!!

Again unexpected pion-kaon differences!

\[ \propto f_{1T}^L \otimes D_1 \]


consistent with Sivers func. of opposite sign for u and d quarks

\[ xN f_U(x) \]

\[ xN f_D(x) \]

\[ \text{Anselmino et al., EPJA 3, 2009} \]
The kaon puzzle in Sivers

$\pi^+/K^+$ production dominated by $u$-quarks, but:

$2 \langle \sin(\phi - \phi_s)/ur \rangle$  

0.1 $\pi^+$  

0.2 $K^+$  

$\pi^+ \equiv |ud\rangle$, $K^+ \equiv |us\rangle \rightarrow$ different role of various sea quarks?

Flavor dependence of $k_T$ in fragment.

$\rightarrow$ impact through convolution integral
The kaon puzzle in Sivers

$\pi^+/K^+$ production dominated by $u$-quarks, but:

- $\pi^+ \equiv |ud\rangle$, $K^+ \equiv |us\rangle$ → different role of various sea quarks?

- Flavor dependence of $k_T$ in fragment. → impact through convolution integral

- Each $x$-bin divided into 2 $Q^2$-bins

- Significant deviations observed only at low $Q^2$

- Hint of systematic diff. only for $K^+$

- Higher-twist contrib. for Kaons
Selected results (2)

The worm-gears

![TMDs Diagram]

**TMDs**

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</tr>
</tbody>
</table>

**worm-gears**
Worm-gears $h_{1L}^\perp$ & $g_{1T}$

\[
\frac{d\sigma^h}{dx dy d\phi_s dz d\phi dP_{h\perp}^2} = \frac{\alpha^2 y^2}{x y Q^2 2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left[ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F^{\cos(\phi)}_{UU} + \epsilon \cos(2\phi) F^{\cos(2\phi)}_{UU} \right]
\]

\[
+ \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F^{\sin(\phi)}_{LU} \right]
\]

\[
+ S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F^{\sin(\phi)}_{UL} + \epsilon \sin(2\phi) F^{\sin(2\phi)}_{UL} \right]
\]

\[
+ S_L \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F^{\cos(\phi)}_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F^{\cos(\phi)}_{LU} \right]
\]

\[
+ S_T \left[ \sin(\phi - \phi s) \left( F^{\sin(\phi - \phi s)}_{UT,T} + \epsilon F^{\sin(\phi - \phi s)}_{UT,L} \right) + \epsilon \sin(\phi + \phi s) F^{\sin(\phi + \phi s)}_{UT} + \epsilon \sin(3\phi - \phi s) F^{\sin(3\phi - \phi s)}_{UT} + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi s) F^{\sin(\phi s)}_{UT} + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi s) F^{\sin(2\phi - \phi s)}_{UT} \right]
\]

\[
+ S_T \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \cos(\phi - \phi s) F^{\cos(\phi - \phi s)}_{LT} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi s) F^{\cos(\phi s)}_{LT} + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi s) F^{\cos(2\phi - \phi s)}_{LT} \right]
\]

Probability to find transversely polarized quarks in a longitudinally polarized nucleon

\[ \propto h_{1L}^\perp \otimes H_1^\perp \]
Worm-gears $h_{1L}^\perp$ & $g_{1T}$

\[
\frac{d\sigma}{dx\,dy\,d\phi\,dz\,d\phi\,dP_{h\perp}^2} = \frac{\alpha^2}{x\,y\,Q^2} \frac{y^2}{2\,(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2\,x} \right)
\]

\[
\{(F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon\,(1+\epsilon)} \cos(\phi)F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi)F_{UU}^{\cos(2\phi)})
\]

\[+ \lambda_L \left[ \sqrt{2\epsilon\,(1-\epsilon)} \sin(\phi)F_{LU}^{\sin(\phi)} \right]\]

\[+ S_L \left[ \sqrt{2\epsilon\,(1-\epsilon)} \sin(\phi)F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi)F_{UL}^{\sin(2\phi)} \right]\]

\[+ S_L \lambda_L \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon\,(1-\epsilon)} \cos(\phi)F_{LL}^{\cos(\phi)} \right]\]

\[+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) + \epsilon \sin(\phi + \phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S)F_{UT}^{\sin(3\phi-\phi_S)} + \sqrt{2\epsilon\,(1+\epsilon)} \sin(\phi_S)F_{UT}^{\sin(\phi_S)} \right.
\]

\[+ \sqrt{2\epsilon\,(1+\epsilon)} \sin(2\phi - \phi_S)F_{UT}^{\sin(2\phi-\phi_S)} \left. \right] \}

Probability to find transversely polarized quarks in a longitudinally polarized nucleon

\[\propto h_{1L}^\perp \otimes H_1^\perp\]

Probability to find longitudinally polarized quarks in a transversely polarized nucleon!

- requires interference between wave funct. components that differ by 1 unit of OAM
- related to orbital motion of partons
- Can be accessed in LT DSAs

\[\propto g_{1T} \otimes D_1\]
The sin(2\(\phi\)) amplitude

\[ \propto h_{1L}^+ \otimes H_1^+ \]

**Deuterium target**

Amplitudes consistent with zero for all mesons and for both H and D targets.

Similar observations by COMPASS on deuterium.

**Hydrogen target**

CLAS reported significant amplitudes for pions on a proton target.

The $\cos(\phi-\phi_S)$ amplitudes

\[ \propto g_{1T} \otimes D_1 \]

- $\pi^+$: slightly positive?
- $\pi^0$: consistent with zero
- $\pi^-$: positive!!
- Similar observations from Hall-A and COMPASS
- $K^+$: slightly positive?
- $K^-$: consistent with zero
Selected results (3)

The higher-twist $F_{LU}^{\sin \phi}$ term
The higher-twist $F_{LU} \sin \phi$ term

$$\frac{d\sigma^h}{dx \, dy \, d\phi \, dz \, dP_{h \perp}^2} = \frac{\alpha^2}{x y Q^2} \frac{y^2}{2 \, (1 - \varepsilon)} \left( 1 + \frac{\gamma^2}{2 \, x} \right) \left\{ \right. $$

$$F_{UU, T} + \varepsilon F_{UU, L} $$

$$+ \sqrt{2\varepsilon} \, (1 + \varepsilon) \cos (\phi) F_{UU}^{\cos} \left( \phi \right) + \varepsilon \cos (2\phi) F_{UU}^{\cos} (2\phi) \right. $$

$$+ \lambda_l \left[ \sqrt{2\varepsilon} \, (1 - \varepsilon) \sin (\phi) F_{LU}^{\sin (\phi)} \right]$$

$$+ S_L \left[ \sqrt{2\varepsilon} \, (1 + \varepsilon) \sin (\phi) F_{UL}^{\sin (\phi)} + \varepsilon \sin (2\phi) F_{UL}^{\sin (2\phi)} \right]$$

$$+ S_T \left[ \sin (\phi - \phi_S) \left( F_{UT, T}^{\sin (\phi - \phi_S)} + \varepsilon F_{UT, L}^{\sin (\phi - \phi_S)} \right) \right.$$

$$+ \varepsilon \sin (\phi + \phi_S) F_{UT}^{\sin (\phi + \phi_S)} + \varepsilon \sin (3\phi - \phi_S) F_{UT}^{\sin (3\phi - \phi_S)}$$

$$+ \sqrt{2\varepsilon} \, (1 + \varepsilon) \sin (\phi_S) F_{UT}^{\sin (\phi_S)}$$

$$+ \sqrt{2\varepsilon} \, (1 + \varepsilon) \sin (2\phi - \phi_S) F_{UT}^{\sin (2\phi - \phi_S)} \left. \right) \}$$

Sensitive to $f_1$, Boer-Mulders + higher-twist DF and FF

$$\propto + \frac{1}{Q} \left[ e \otimes H_{1 \perp}^T + f_1 \otimes \tilde{G}_{1 \perp} \right.$$  

$$+ g_{1 \perp} \otimes D_{1} + h_{1 \perp} \otimes \tilde{E} \right]$$
The $F_{LU} \sin \phi$ term

$$\propto \frac{1}{Q} \left[ e \otimes H_+^1 + f_1 \otimes \tilde{G}^\perp + g_1 \otimes D_1 + h_1 \otimes E \right]$$

H target, 2000-2007 data 0.2<z<0.7

Amplitudes are positive for pions and consistent with zero for kaons and protons
The $F_{LU} \sin \phi$ term

$$\alpha + \frac{1}{Q} \left[ e \otimes H_1^\perp + f_1 \otimes \tilde{G}^\perp + g^\perp \otimes D_1 + h_1^\perp \otimes \tilde{E} \right]$$

D target, 2000-2007 data 0.2<z<0.7

Amplitudes are positive for pions and consistent with zero for kaons and protons
Deuterium target: same features, less statistics
Part II

Inclusive electroproduction of hadrons
From SIDIS to inclusive hadron production

**SIDIS:** $lp^\uparrow \rightarrow l'hX$

- Hadron detected in coincidence with lepton
- DIS regime ($Q^2 > 1 \text{ GeV}^2$)
- Hard scales: $Q^2, P_{h\perp}$ (w.r.t. $\gamma^*$)
- Factorization valid for $P_{h\perp}^2 \ll Q^2$
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**Inclusive hadrons:**  $lp^\uparrow \rightarrow hX$
- **Lepton is not detected**  $\rightarrow$ no info on $Q^2$
- data dominated by $Q^2 \approx 0$
  (quasi-real photoproduction regime)
- Hard scales: $P_T$ (w.r.t. incident lepton)
- Factorization valid for large $P_T$?
- Main variables: $x_F = 2 \frac{P_L}{\sqrt{s}}$, $P_T$
- **Selected events** contain at least 1 charged hadron track ($\pi$ or K) regardless of whether there was also a scattered lepton in acceptance or not.
From SIDIS to inclusive hadron production

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- Factorization valid for large \( P_T \)?
- Main variables: \( x_F = 2 \frac{P_L}{\sqrt{s}} , P_T \)
- **Selected events** contain at least 1 charged hadron track (\( \pi \) or \( K \)) regardless of whether there was also a scattered lepton in acceptance or not.
- **SIDIS events constitute a small subsample**

---

**Hadron yields for UT data**

<table>
<thead>
<tr>
<th></th>
<th>( \pi^+ )</th>
<th>( \pi^- )</th>
<th>( K^+ )</th>
<th>( K^- )</th>
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<tbody>
<tr>
<td>SIDIS</td>
<td>7.3 M</td>
<td>5.4 M</td>
<td>131 K</td>
<td>54 K</td>
</tr>
<tr>
<td>Incl. h</td>
<td>60 M</td>
<td>50 M</td>
<td>5.1 M</td>
<td>2.8 M</td>
</tr>
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</table>
Cross section and azimuthal asymmetries

\[ d\sigma = d\sigma_{UU} [1 + S_\perp A_{UT} \sin \psi \sin \psi] \]

\[ \vec{S} \cdot (\vec{P}_h \times \vec{k}) \propto \sin \psi \]

\( \psi \): azimuthal angle between the upwards target spin direction and hadron production plane around the beam direction
Cross section and azimuthal asymmetries

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\[ \vec{s} \cdot (\vec{P}_h \times \vec{k}) \propto \sin \psi \]

\( \psi \): azimuthal angle between the upwards target spin direction and hadron production plane around the beam direction

For an ideal detector with full \(2\pi\) coverage in \(\psi\):

\[ A_{sin \psi}^{\sin \psi} = - \frac{\pi}{2} \cdot \frac{\int_0^\pi d\psi \ d\sigma_{UT} \ \sin \psi}{\int_0^\pi d\psi \ d\sigma_{UT}} = - \frac{\pi}{2} A_N \]
Cross section and azimuthal asymmetries

\[ d\sigma = d\sigma_{UU}[1 + S_{\perp}A_{UT}\sin\psi \sin\psi] \]

\[ \vec{s} \cdot (\vec{P}_h \times \vec{k}) \propto \sin\psi \]

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Polarized pp scattering experiments observe asymmetries up to 40%!

- mirror symmetric for \(\pi^+\) and \(\pi^-\) vs. \(x_F\)
- reproduced by various exp. over 35 years, persistent with energy (\(\sqrt{s}\) from 5 to 200 GeV !)
- Cannot be interpreted using the standard leading-twist framework based on collinear factorization

Inclusive hadrons results

\[ A_{UT}^{\pi^+} \] amplitude rises linearly with \( x_F \) up to 10%

\[ A_{UT}^{\pi^-} \] is negative, similar trend, smaller (up to 4%)
Inclusive hadrons results

\[ \pi^+ \text{ amplitude rises linearly with } x_F \text{ up to 10%} \]
\[ \pi^- \text{ is negative, similar trend, smaller (up to } 4\%) \]

General trend very similar to \( A_N \) in \( pp^\uparrow \) hard scattering

- \( A_N \) in \( p^\uparrow p \) scattering is much larger and mirror symmetric for \( \pi^+ \) and \( \pi^- \)
- \( u \)-quark dominance in \( ep^\uparrow \) scattering can explain the relatively smaller size for \( \pi^- \)
**Inclusive hadrons results**

$\pi^+$ amplitude rises linearly with $x_F$ up to 10%

$\pi^-$ is negative, similar trend, smaller (up to 4%)

$K^+$ is about constant around 7%

$K^-$ ≈ 0

Again kaon behave differently than pions!

General trend very similar to $A_N$ in $pp^\uparrow$ hard scattering

- $A_N$ in $p^\uparrow p$ scattering is much larger and mirror symmetric for $\pi^+$ and $\pi^-$

- $u$-quark dominance in $ep^\uparrow$ scattering can explain the relatively smaller size for $\pi^-$
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\( \pi^+ \) amplitude rises linearly with \( x_F \) up to 10%
\( \pi^- \) is negative, similar trend, smaller (up to 4%)
\( K^+ \) is about constant around 7%
\( K^- \approx 0 \)

Again kaon behave differently than pions!

\( \pi^+ \) and \( K^+ \) amplitudes rise linearly up to
\( P_T \approx 0.8 \text{ GeV} \) then decrease with increasing \( P_T \)
\( \pi^+ \) also show a clear rise above \( P_T \approx 1.3 \text{ GeV} \)

Amplitudes of negative mesons are much smaller apart for a \( \pi^- \) point at \( P_T \approx 1.5 \text{ GeV} \)
Inclusive hadrons results

\[ \pi^+ \text{ amplitude rises linearly with } x_F \text{ up to 10\%} \]
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\[ K^+ \text{ is about constant around 7\%} \]
\[ K^- \approx 0 \]

Again kaon behave differently than pions!

\[ x_F \text{ and } P_T \text{ strongly correlated.} \]

Important to look at 2D extractions

\[ \pi^+ \text{ and } K^+ \text{ amplitudes rise linearly up to } P_T \approx 0.8 \text{ GeV} \text{ then decrease with increasing } P_T \]

\[ \pi^+ \text{ also show a clear rise above } P_T \approx 1.3 \text{ GeV} \]

Amplitudes of negative mesons are much smaller apart for a \( \pi^- \) point at \( P_T \approx 1.5 \text{ GeV} \)
Inclusive hadrons results

\[ A_{\sin \psi}^{+} \]

- \( 0.00 < x_F < 0.10 \)
- \( 0.10 < x_F < 0.20 \)
- \( 0.20 < x_F < 0.30 \)
- \( 0.30 < x_F < 0.55 \)

\[ A_{\sin \psi}^{+} \]

- \( 0.66 < P_t [\text{GeV}] < 1.00 \)
- \( 1.00 < P_t [\text{GeV}] < 2.20 \)

8.8% scale uncertainty

\[ P_t [\text{GeV}] \]

\[ x_F \]
Inclusive hadrons results

- $\pi^+$ amplitudes vs. $P_T$ are basically the same in all $x_F$ bins $\rightarrow$ apparent increase in magnitude vs. $x_F$ in 1D projections is a reflection of underlying dependence on $P_T$
- $\pi^-$ amplitudes vs. $P_T$ are vanishing at low $x_F$ and become negative at high $x_F$
Interpretation

- The inclusive hadron electroproduction data set is a mixture of various contributions with different kinematic dependences difficult to draw conclusions on the underlying physics from the observed kinematic dependences.
- More insight may be gained by studying separately the asymmetries for different subsamples.

Inclusive hadron production (all events)

Quasi-real photoproduction ($Q^2 \approx 0$)

- Quasi-real photoproduction ($Q^2 \approx 0$)
- Identified lepton events without scattered lepton in acceptance ("anti-tagged")

DIS $0.2 < z < 0.7$ (SIDIS)

DIS $z > 0.7$ (Exclusive)
Interpretation

• The inclusive hadron electroproduction data set is a mixture of various contributions with different kinematic dependences → difficult to draw conclusions on the underlying physics from the observed kinematic dependences

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Inclusive hadron production
(all events)

Quasi-real photoproduction
\( (Q^2 \approx 0) \)

- Anti-tagged:
  - About 98% of total statistics → asymmetries vs. \( P_T \) essentially identical to inclusive amplitudes at low-to-intermediate \( P_T \).
  - For \( P_T > 1.3 \, GeV \) they differ due to the contributions from the other subsamples to the full inclusive sample

\[ \text{Anti-tagged} \]

- Identified lepton
  - DIS \( 0.2 < z < 0.7 \) (SIDIS)
  - DIS \( z > 0.7 \) (Exclusive)
Interpretation

DIS $0.2 < z < 0.7$:
- $\pi^+/\pi^-$ amplitudes larger than inclusive in full $P_T$ range and rise linearly with $P_T$ (up to 20% for $\pi^+$)
- In this regime $Q^2 > P_T^2$ and TMDs can contribute without $P_T$-suppression
- Since $\psi$ and $\phi - \phi_S$ are closely related the observed $P_T$ dependence might arise from the Sivers effect

Anti-tagged:
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**DIS \(0.2 < z < 0.7\):**
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- Since \(\psi\) and \(\phi - \phi_S\) are closely related the observed \(P_T\) dependence might arise from the Sivers effect

**DIS \(z > 0.7\):**
- Very large asymmetries observed for pions and especially \(K^+\) (more than 40%)
- Pions receive large contributions from decays of exclusive \(\rho\)
- \(\pi^-\) large amplitude may come from d-quark Sivers function in conjunction with favored fragmentation of the struck (down) quark

### Quasi-real photoproduction (\(Q^2 \approx 0\))

- DIS \(0.2 < z < 0.7\) (SIDIS)
- DIS \(z > 0.7\) (Exclusive)

**Anti-tagged:**
- About 98% of total statistics \(\rightarrow\) asymmetries vs. \(P_T\) essentially identical to inclusive amplitudes at low-to-intermediate \(P_T\).
- For \(P_T > 1.3\) GeV they differ due to the contributions from the other subsamples to the full inclusive sample
Conclusions

A rich phenomenology and surprising effects arise when intrinsic $p_T$ is not integrated out!

Flavor sensitivity ensured by the excellent hadron ID revealed interesting facets of data.
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The HERMES experiment has played a pioneering role in these studies:
Conclusions

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The HERMES experiment has played a pioneering role in these studies:

HERMES results in inclusive hadron electroproduction reveal interesting features in common with $A_N$ in $pp^\uparrow$ scattering and with Sivers effect in SIDIS.

A rich phenomenology is revealed when the various subsamples are analyzed separately.
Back-up
Transversity

\[
\frac{d\sigma^h}{dx \, dy \, d\phi \, dS \, dz \, d\phi \, dP^2_{hL}} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \varepsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right\}
\]

\[
+ \lambda L \left[ \sqrt{2\varepsilon(1-\varepsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]
\]

\[
+ S_L \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \varepsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]
\]

\[
+ S_L \lambda L \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]
\]

\[
+ S_T \left[ \sin(\phi - \phi S) \left(F_{UT,T}^{\sin(\phi - \phi S)} + \varepsilon F_{UT,L}^{\sin(\phi - \phi S)} \right) + \varepsilon \sin(\phi + \phi S) F_{UT}^{\sin(\phi + \phi S)} + \varepsilon \sin(3\phi - \phi S) F_{UT}^{\sin(3\phi - \phi S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(\phi S) F_{UT}^{\sin(\phi S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi - \phi S) F_{UT}^{\sin(2\phi - \phi S)} \right]
\]

\[
+ S_T \lambda L \left[ \sqrt{1-\varepsilon^2} \cos(\phi - \phi S) F_{LT}^{\cos(\phi - \phi S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(\phi S) F_{LT}^{\cos(\phi S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi - \phi S) F_{LT}^{\cos(2\phi - \phi S)} \right] \right\}
\]

Describes probability to find transversely polarized quarks in a transversely polarized nucleon.
Collins amplitudes $\propto h_1(x, p_T^2) \otimes H_1(z, k_T^2)$

Consistent with Belle/BaBar measurements in $e^+e^-$

\[ \frac{u \rightarrow \pi^-}{d \rightarrow \pi^+} \quad \frac{u \rightarrow \pi^+}{d \rightarrow \pi^-} \]

\[ H_{1,un}^{f, av}(z) \approx H_{1, f}^{av}(z) \]

Consistent with isospin-symmetry

\[ e^+e^- \rightarrow \pi^+_{jet1} \pi^-_{jet2} X \]

Positive

Consistent with zero (isospin-symmetry)

Large and negative!

Significantly positive

Consistent with zero
Subleading twist

\[
\frac{d\sigma^h}{dx dy d\phi_S d\phi dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left( 1 + \frac{\gamma^2}{2x} \right)
\]

\[
\left\{ \begin{array}{l}
F_{UU,T} + \epsilon F_{UU,L} \\
+ \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \\
+ \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\
+ S_L \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \\
+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\
+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) \\
+ \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \\
+ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\
+ \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right] \\
+ S_T \lambda_l \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \\
+ \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\
+ \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right] \end{array} \right\}
\]

\[F_{UT}^{\sin}\phi_S = \frac{2M}{Q} C \left\{ \left( x f_T D_1 - \frac{M_h}{M} h_1 \tilde{H}_1 \right) \\
- \frac{k_T \cdot p_T}{2MM_h} \left[ \left( x h_T H_{1T}^\perp + \frac{M_h}{M} g_{1T} \tilde{G}_{1T} \right) - \left( x h_T H_{1T}^\perp - \frac{M_h}{M} f_{1T} \tilde{D}_{1T} \right) \right] \right\} \]

Sensitive to worm-gear \( g_{1T}^\perp \), sivers, transversity + higher-twist DF and FF
Subleading-twist $\sin(\phi_S)$ Fourier component

- sensitive to worm-gear $g_{1T}$, Sivers function, Transversity, etc
- significant non-zero signal for $\pi^-$ and $K^-$!

![Graph showing $2\langle \sin(\phi_S) \rangle_{u_1}$ for different particles and data points highlighting large and negative behavior for $\pi^-$ and $K^-$, along with a hint of $Q^2$ dependence for $\pi^-$ with low-$Q^2$ amplitude larger]
Pretzelosity

\[ \frac{d\sigma^h}{dx \, dy \, d\phi \, d\rho \, d\mathbf{P}_{h \perp}^2} = \frac{\alpha^2 \, y^2}{x \, y \, Q^2 \, 2 \, (1 - \epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ \begin{array}{l} F_{UU,T} + \epsilon F_{UU,L} \\
\sqrt{2\epsilon \, (1 + \epsilon)} \cos(\phi) F_{UU}^{cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{cos(2\phi)} \\
+ \lambda_l \sqrt{2\epsilon \, (1 - \epsilon)} \sin(\phi) F_{LU}^{sin(\phi)} \end{array} \right\} \]

\[ + \left\{ \begin{array}{l} S_L \left[ \sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2\epsilon \, (1 - \epsilon)} \cos(\phi) F_{UL}^{cos(\phi)} \right] \\
+ S_L \lambda_l \left[ \sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2\epsilon \, (1 - \epsilon)} \cos(\phi) F_{UL}^{cos(\phi)} \right] \end{array} \right\} \]

\[ + S_T \left[ \sin(\phi - \phi_s) \left( F_{UT,T}^{sin(\phi - \phi_s)} + \epsilon F_{UT,L}^{sin(\phi - \phi_s)} \right) \\
+ \epsilon \sin(\phi + \phi_s) F_{UT}^{sin(\phi + \phi_s)} + \epsilon \sin(3\phi - \phi_s) F_{UT}^{sin(3\phi - \phi_s)} \\
+ \sqrt{2\epsilon \, (1 + \epsilon)} \sin(\phi_s) F_{UT}^{sin(\phi_s)} \\
+ \sqrt{2\epsilon \, (1 + \epsilon)} \sin(2\phi - \phi_s) F_{UT}^{sin(2\phi - \phi_s)} \right] \]

\[ + S_T \lambda_l \left[ \sqrt{1 - \epsilon^2} \cos(\phi - \phi_s) F_{LT}^{cos(\phi - \phi_s)} \\
+ \sqrt{2\epsilon \, (1 - \epsilon)} \cos(\phi_s) F_{LT}^{cos(\phi_s)} \\
+ \sqrt{2\epsilon \, (1 - \epsilon)} \cos(2\phi - \phi_s) F_{LT}^{cos(2\phi - \phi_s)} \right] \}

Describes correlation between quark transverse momentum and transverse spin in a transversely pol. nucleon

➢ Sensitive to non-spherical shape of the nucleon

### Distribution Functions

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<tr>
<th>Quark</th>
<th>U</th>
<th>L</th>
<th>T</th>
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<tr>
<td>U</td>
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<td>( g_1 )</td>
<td>( h_{1T} )</td>
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### Fragmentation Functions

<table>
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<td>U</td>
<td>( D_1 )</td>
<td>( H_{1\perp} )</td>
<td>( D_1 )</td>
</tr>
</tbody>
</table>

The $\sin(3\phi - \phi_S)$ amplitude $\propto h_{1T}(x, p_T^2) \otimes H_1(z, k_T^2)$

All amplitudes consistent with zero

...suppressed by two powers of $P_{h\perp}$ w.r.t. Collins and Sivers amplitudes
The worm-gear $g_{1T}^\perp$

- The only TMD that is both chiral-even and naïve-T-even
- Requires interference between wave funct. components that differ by 1 unit of OAM

$g_{1T}^\perp$ (also supported by Lattice QCD and first data)

Many models support simple relations among $g_{1T}^\perp$ and other TMDs:

- $g_{1T}^q = -h_{1L}^q$
- $g_{1T}^{(1)} (x) \approx \int x \frac{dy}{y} g_{1}^q (y)$ (Wandzura-Wilczek appr.)

S. Boffi et al. (2009)
Phys. Rev. D 79 094012
Light cone constituent quark model
flavorless
dashed line: interf. L=0, L=1
dotted line: interf L=1, L=2

⇒ related to quark orbital motion inside nucleons

Related to quark orbital motion inside nucleons
Probing $g_{1T}$ through Double Spin Asymmetries

\[ F_{LT}^{\cos(\phi_h - \phi_S)} = C \left[ \frac{\hat{h} \cdot p_T}{M} g_{1T} D_1 \right] \]

\[ F_{LT}^{\cos \phi_S} = \frac{2M}{Q} C \left\{ - \left( x g_T D_1 + \frac{M_h}{M} h_1 \frac{\tilde{E}}{z} \right) \right. \]
\[ \left. + \frac{k_T \cdot p_T}{2MM_h} \left[ \left( x e_T H_1^T - \frac{M_h}{M} g_{1T} \frac{\tilde{D}}{z} \right) + \left( x e_T H_1^T + \frac{M_h}{M} f_{1T} \frac{\tilde{G}}{z} \right) \right] \right\} \]

\[ F_{LT}^{\cos (2\phi_h - \phi_S)} = \frac{2M}{Q} C \left\{ \frac{2 (\hat{h} \cdot p_T)^2 - p_T^2}{2M^2} \left( x g_T D_1 + \frac{M_h}{M} h_1 \frac{\tilde{E}}{z} \right) \right. \]
\[ \left. + \frac{2 (\hat{h} \cdot k_T) (\hat{h} \cdot p_T) - k_T \cdot p_T}{2MM_h} \left[ \left( x e_T H_1^T - \frac{M_h}{M} g_{1T} \frac{\tilde{D}}{z} \right) \right. \right. \]
\[ \left. \left. \left. - \left( x e_T H_1^T + \frac{M_h}{M} f_{1T} \frac{\tilde{G}}{z} \right) \right] \right\} \]

The simplest way to probe worm-gear $g_{1T}$ is through the $\cos(\phi - \phi_S)$ Fourier component
The $\cos(\phi_S)$ Fourier component

$2 \langle \cos(\phi_S) \rangle_{L}^\pi$

$\approx 0$

$2 \langle \cos(\phi_S) \rangle_{L}^{\pi^0}$

$\approx 0$

$2 \langle \cos(\phi_S) \rangle_{L}^{\pi^-}$

$\approx 0$

$2 \langle \cos(\phi_S) \rangle_{L}^{K^+}$

$\approx 0$

$2 \langle \cos(\phi_S) \rangle_{L}^{K}$

$\approx 0$

sign change?
The $\cos(2\phi-\phi_S)$ Fourier component
The $\sin(2\phi + \phi_S)$ Fourier component

- arises solely from longitudinal (w.r.t. virtual photon direction) component of the target spin

- related to $\langle \sin(2\phi) \rangle_{UL}$ Fourier comp:
  \[ 2\langle \sin(2\phi + \phi_S) \rangle_{UL}^h \propto \frac{1}{2} \sin(\mathcal{G}_{y^*})2\langle \sin(2\phi) \rangle_{UL}^h \]

- sensitive to worm-gear $h_{1L}^\perp$

- suppressed by one power of $P_{h_L}$ w.r.t. Collins and Sivers amplitudes

- no significant signal observed (except maybe for $K^+$)
The subleading-twist $\sin(2\phi-\phi_S)$ Fourier component

- sensitive to **worm-gear** $g_{1T}^\perp$, Pretzelosity and Sivers function:

$$\propto W_1(p_T, k_T, P_{h\perp}) \left( x f_T^+ D_1 - \frac{M_h}{M} h_{1T}^+ \tilde{H} \right)$$

$$- W_2(p_T, k_T, P_{h\perp}) \left[ x h_T H_1^+ + \frac{M_h}{M} g_{1T}^+ \tilde{G}^\perp \right]$$

$$+ \left( x h_T H_1^+ - \frac{M_h}{M} f_{1T}^+ \tilde{D}^\perp \right)$$

- suppressed by one power of $P_{h\perp}$ w.r.t. Collins and Sivers amplitudes

- no significant non-zero signal observed
\[ F_{LU} \sin \phi \]

\[
\frac{d\sigma^h}{dx \, dy \, d\phi_S \, dz \, d\phi \, dP_{h \perp}^2} = \alpha^2 \frac{y^2}{xyQ^2} \frac{1 + \gamma^2}{2x} \left( 1 + \gamma^2 \right) \\
\left\{ \begin{array}{l}
F_{UU,T} + \epsilon F_{UU,L} \\
+ \sqrt{2\epsilon} \frac{(1+\epsilon)}{\cos(\phi)} F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \\
+ \lambda_l \left[ \sqrt{2\epsilon} \frac{(1-\epsilon)}{\sin(\phi)} F_{LU}^{\sin(\phi)} \right] \\
+ S_L \left[ \sqrt{2\epsilon} \frac{(1+\epsilon)}{\sin(\phi)} F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\
+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon} \frac{(1-\epsilon)}{\cos(\phi)} F_{LL}^{\cos(\phi)} \right] \\
+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) \\
+ \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \\
+ \sqrt{2\epsilon} \frac{(1+\epsilon)}{\sin(\phi_S)} F_{UT}^{\sin(\phi_S)} \\
+ \sqrt{2\epsilon} \frac{(1-\epsilon)}{\sin(2\phi - \phi_S)} F_{UT}^{\sin(2\phi - \phi_S)} \right] \\
+ S_T \lambda_l \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \\
+ \sqrt{2\epsilon} \frac{(1-\epsilon)}{\cos(\phi_S)} F_{LT}^{\cos(\phi_S)} \\
+ \sqrt{2\epsilon} \frac{(1-\epsilon)}{\cos(2\phi - \phi_S)} F_{LT}^{\cos(2\phi - \phi_S)} \right] \right\} 
\]

\[ F_{LU}^{\sin\phi_h} = \frac{2M}{Q} \frac{\epsilon \cdot k_T}{M_h} \left( x e H_T^L + \frac{M_h}{M} f_1 \frac{G_T^L}{z} \right) + \frac{\epsilon \cdot p_T}{M} \left( x g D_1 + \frac{M_h}{M} h_1 \frac{E}{z} \right) \]
The HERMES experiment at HERA

TRD, Calorimeter, preshower, RICH: lepton-hadron > 98%

hadron separation

Aerogel $n=1.03$

$C_4F_{10}$ $n=1.0014$

$\pi \sim 98\%, K \sim 88\%, P \sim 85\%$
2-hadron SIDIS results

Following formalism developed by Steve Gliske

Find details in

Transverse Target Moments of Dihadron Production in Semi-inclusive Deep Inelastic Scattering at HERMES
S. Gliske, PhD thesis, University of Michigan, 2011

http://www-personal.umich.edu/~lorenzon/research/HERMES/PHDs/Gliske-PhD.pdf
A short digression on di-hadron fragmentation functions

**Standard definition** of di-hadron FF assume no polarization of final state hadrons (pseudo-scalar mesons) or define mixtures of certain partial waves as new FFs.

In the **new formalism** there are only two di-hadron FFs. Names and symbols are entirely associated with the quark spin, whereas the partial waves of the produced hadrons ($|l_1 m_1>, |l_2 m_2>$) are associated with partial waves of FFs.

\[
\chi = \chi' \quad \Rightarrow \quad D_1 = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos \vartheta) e^{im(\phi_R - \phi_k)} D_1^{(\ell,m)}(z, M_h, |k_T|)
\]

\[
\chi \neq \chi' \quad \Rightarrow \quad H_1^{\perp} = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos \vartheta) e^{im(\phi_R - \phi_k)} H_1^{\perp(\ell,m)}(z, M_h, |k_T|)
\]

The cross-section is identical to the ones in literature, the only difference is the interpretation of the FFs:

\[
D_1^{(0,0)} = D_{1,OO} = \left(\frac{1}{4} D_{1,OO}^s + \frac{3}{4} D_{1,OO}^p\right)
\]

\[
D_1^{(1,0)} = D_{1,OL}
\]

\[
D_1^{(1,\pm 1)} = D_{1,OT} \pm \frac{|k_T||R|}{M_h^2} G_{1,OT}^{\perp}
\]

\[
D_1^{(2,0)} = \frac{1}{2} D_{1,LL}
\]

\[
D_1^{(2,\pm 1)} = \frac{1}{2} \left(D_{1,LT} \pm \frac{|k_T||R|}{M_h^2} G_{1,LT}^{\perp}\right)
\]

\[
D_1^{(2,\pm 2)} = D_{1,TT} \pm \frac{1}{2} \frac{|k_T||R|}{M_h^2} G_{1,TT}^{\perp}
\]

\[
H_1^{(0,0)} = H_{1,OO}^{\perp} = \frac{1}{4} H_{1,OO}^s + \frac{3}{4} H_{1,OO}^p
\]

\[
H_1^{(1,1)} = H_{1,OT}^{\perp} + \frac{|R|}{|k_T|} H_{1,OT}^{\perp}
\]

\[
H_1^{(0,0)} = H_{1,OO}^{\perp} = \frac{1}{4} H_{1,OO}^s + \frac{3}{4} H_{1,OO}^p
\]

\[
H_1^{(2,0)} = \frac{1}{2} H_{1,LL}^{\perp}
\]

\[
H_1^{(2,-1)} = \frac{1}{2} H_{1,LT}^{\perp}
\]

\[
H_1^{(2,-2)} = H_{1,TT}^{\perp}
\]
The di-hadron SIDIS cross-section

\[ d\sigma_{UT} = \frac{\alpha^2 M_h P_{h\perp}}{2\pi x y Q^2} \left( 1 + \frac{\gamma^2}{2x} \right) |S_\perp| \]

\[ \times \sum_{\ell=0}^{2} \sum_{m=-\ell}^{\ell} \left\{ A(x, y) \left[ P_{\ell,m} \sin((m + 1)\phi_h - m\phi_R - \phi_S) \right] \right. \]

\[ \left. \times \left( F_{UT,T}^{P_{\ell,m}\sin((m+1)\phi_h-m\phi_R-\phi_S)} + \epsilon F_{UT,L}^{P_{\ell,m}\sin((m+1)\phi_h-m\phi_R-\phi_S)} \right) \right\} \]

\[ + B(x, y) \left[ P_{\ell,m} \sin((1 - m)\phi_h + m\phi_R + \phi_S) \right] \]

\[ + P_{\ell,m} \sin((3 - m)\phi_h + m\phi_R - \phi_S) F_{UT}^{P_{\ell,m}\sin((3-m)\phi_h+m\phi_R-\phi_S)} \]

\[ + V(x, y) \left[ P_{\ell,m} \sin(-m\phi_h + m\phi_R + \phi_S) F_{UT}^{P_{\ell,m}\sin(-m\phi_h+m\phi_R+\phi_S)} \right. \]

\[ \left. + P_{\ell,m} \sin((2 - m)\phi_h + m\phi_R - \phi_S) F_{UT}^{P_{\ell,m}\sin((2-m)\phi_h+m\phi_R-\phi_S)} \right\} \}

\[ l \text{ and } m \text{ correspond to } |lm\rangle \text{ angular momentum state of the hadron} \]

Considering all terms \((d\sigma_{UU}, d\sigma_{LU}, d\sigma_{UL}, d\sigma_{LL}, d\sigma_{UT}, d\sigma_{LT})\) there are 144 non-zero structure functions at twist-3 level. The most known is

\[ F_{UT}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_R+\phi_S)} = -\mathcal{I} \left[ \frac{|k_T|}{M_h} \cos \left( (m - 1)\phi_h - \phi_p - m\phi_k \right) h_1 H_1^{\perp \ell, m} \right] \]

which for \(l = 1\) and \(m = 1\) reduces to the well known collinear \(F_{UT} \sin^3 \theta \sin(\phi_R + \phi_S)\) related to transversity.
The di-hadron SIDIS cross-section

- independent way to access transversity
- Collinear → no convolution integral
- significantly positive amplitudes
- 1\textsuperscript{st} evidence of non zero dihadron FF
- limited statistical power (v.r.t. 1 hadron)

\[ \sigma_{LT} \propto S_T \sin \theta \sin \phi_\rho \sum_q e_q^2 \delta \phi H_{\gamma q} \]
The di-hadron SIDIS cross-section

- independent way to access transversity
- Collinear → no convolution integral
- significantly positive amplitudes
- $1^{st}$ evidence of non zero dihadron FF
- limited statistical power (v.r.t. 1 hadron)
- signs are consistent for all $\pi\pi$ species
- statistics much more limited for $\pi^{\pm}\pi^{0}$
- despite uncertainties may still help to constrain global fits and may assist in $u-d$ flavor separation

JHEP 06 (2008) 017

Published $\pi^{+}\pi^{-}$ Results

New $\pi^{\pm}\pi^{0}$ Results

- New tracking, new PID, use of $\phi_{R}$ rather than $\phi_{R\perp}$
- Different fitting procedure and function
- Acceptance correction
A short digression on the Lund/Artru string fragmentation model
(a phenomenological explanation of the Collins effect)

In the cross-section the Collins FF is always paired with a distribution function involving a transverse polarized quark.

1. Assume u quark and proton have (transverse) spin aligned in the direction $\phi_s = \pi/2$. The model assumes that the struck quark is initially connected with the remnant via a gluon-flux tube (string).

2. When the string breaks, a $q\bar{q}$ pair is created with vacuum quantum numbers $J^P = 0^+$. The positive parity requires that the spins of $q$ and $\bar{q}$ are aligned, thus an OAM $L = 1$ has to compensate the spins.

3. This OAM generates a transverse momentum of the produced pseudo-scalar meson (e.g. $\pi^+$) and deflects the meson to the left side w.r.t. the struck quark direction, generating left-right azimuthal asymmetries.
A short digression on the Lund/Artru string fragmentation model

Relative to the proton transv. spin, the fragmenting quark can have spin parallel or antiparallel to \( \frac{1}{2}, \pm \frac{1}{2} \).

Then combining the spins of the formed di-quark systems one can get:

\[
\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0 \Rightarrow \begin{cases} 
1 \text{ spin 0 state } |0,0\rangle & 1 \text{ pseudo-scalar meson (PSM)} \\
3 \text{ spin 1 states} \{ |1,0\rangle & 1 \text{ Longitudinal VM} \\
& |1,\pm 1\rangle & 2 \text{ transvers VM} 
\end{cases}
\]

**Lund/Artru prediction at the amplitude level**: the asymmetry for PSM has opposite sign to that for transversely polarized VM (left vs. right side), and the amplitude for \( |1,0\rangle \) is 0.

Lund/Artru model makes predictions for the individual di-hadrons, but the Collins function includes pairs of di-hadrons.

\( \rightarrow \) to make predictions for the Collins function one needs to consider the cross-section level, i.e. the sum of contributing amplitudes times their complex conjugate.

Using the Clebsch-Gordan algebra one obtains: \( |1,\pm 1\rangle |1,\pm 1\rangle \equiv |2,\pm 2\rangle \)

**Lund/Artru prediction at the cross-section level**: the \( |2,\pm 2\rangle \) partial waves of the Collins func. for SIDIS VM production have the opposite sign as the respective PS Collins func.
“gluon radiation model” vs. Lund/Artru model

The Lund/Artru model only accounts for favored Collins fragmentation. An extension of the model (the **gluon radiation model**), elaborated by S. Gliske accounts for the disfavored case

1. Struck quark emits a gluon in such a way that most of its momentum is transferred to the gluon
2. The struck quark then becomes part of the remnant
3. The radiated gluon produces a $q\bar{q}$ pair that eventually converts into a meson
4. For PSM the di-quark must interact further with the remnant to get the PSM quantum numbers. In case of VM the di-quark directly forms the meson

**Lund/Artu**

- Di-quark has q.n. of vacuum
- **Struck quark** joins the anti-quark in the final state $\rightarrow$ **favored fragment**.

**Gluon radiation**

- Di-quark has q.n. of observed final state
- **Produced quark** joins the anti-quark in the final state $\rightarrow$ **disfavored fragment**.

**Prediction:** the $|2, \pm 2\rangle$ partial wave of the Collins funct. for SIDIS VM production have the opposite sign as the respective PS Collins function

**Prediction:** the disfavored $|2, \pm 2\rangle$ Collins frag. also is expected to have opposite sign as the respective PS Collins function.

Models predict: fav = disfav for VM
Data say: fav $\cong -$ disfav for PSM (Collins $\pi^+$ vs. $\pi^-$)
...and now let’s look at the results

<table>
<thead>
<tr>
<th>Fragment. process</th>
<th>Fav/disfav</th>
<th>Deflection</th>
<th>Sign of amplitude</th>
</tr>
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<tr>
<td>$u \to \pi^+$</td>
<td>fav PSM</td>
<td>left ($\phi_h \to 0$)</td>
<td>$&gt; 0$ (Collins $\pi^+$)</td>
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<tr>
<td>$u \to \pi^-$</td>
<td>disfav PSM</td>
<td>right ($\phi_h \to \pi$)</td>
<td>$&lt; 0$ (Collins $\pi^-$)</td>
</tr>
<tr>
<td>$u \to \rho^+ \to \pi^+\pi^0$</td>
<td>fav VM</td>
<td>right ($\phi_h \to \pi$)</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$u \to \rho^- \to \pi^-\pi^0$</td>
<td>disfav VM</td>
<td>right ($\phi_h \to \pi$)</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$u \to \rho^0 \to \pi^+\pi^-$</td>
<td>mixed VM</td>
<td>right ($\phi_h \to \pi$)</td>
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<td>( &lt; 0 ) (Collins ( \pi^- ))</td>
</tr>
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<td>( &lt; 0 )</td>
</tr>
<tr>
<td>( u \to \rho^- \to \pi^-\pi^0 )</td>
<td>disfav VM</td>
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<td>0 or ( &lt; 0 )</td>
</tr>
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</table>

\(|2, -2\rangle\) consistent with zero for all flavors

Not in contraddiction with models: if the transversity function causes the fragmenting quark to have positive polarization than Collins \(|2, -2\rangle\) must be zero as this partial wave requires fragmenting quark with negative polarization

\(|2, +2\rangle\) consistent with model expect:
- No signal outside \( \rho\)-mass bin
  \( \to \) no non-resonant pion-pairs in p-wave
- Negative for \( \rho^\pm \) (model predictions)
- very small for \( \rho^0 \) (consistent with small Collins \( \pi^0 \) )
Assume scattering off a $u$ quark

...in next 5 slides a *naive* representation of a fragmentation process that can lead to protons/antiprotons in the final states
Proton production

\[ e \rightarrow e' \gamma^* \rightarrow u + d \bar{d} \]
Proton production

\[ e + \gamma^* \rightarrow e' + \bar{d} + \bar{u} + \bar{u} + \bar{d} + \bar{s} + s \]
Proton production

\[ u + \ldots \]

\[ \bar{d} + \bar{u} \]

\[ d + \ldots \]

\[ u + \ldots \]

\[ \bar{u} + \bar{d} \]

\[ s + \ldots \]

\[ = p + \bar{p} + \bar{\Lambda} + \ldots ? \]

\[ \bar{p} + \pi^+ \]
Proton production

\[
\gamma^* \rightarrow e^+e^- \rightarrow u + d + \bar{u} + \bar{d} + s + ... \Rightarrow p + \bar{p} + \bar{\Lambda} + ... ? \\
\bar{p} + \pi^+
\]

...but also from target fragmentation