Ratios of Helicity Amplitudes for Exclusive $\rho^0$ Electroproduction

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In the scattering process depicted in Fig. 1, an electron scatters from a nucleon target. The process produces a meson in the final state and leaves the target intact. This type of process can be useful for discovering detail about the structure of the target proton; by measuring the details of the produced meson, the quark structure of the proton can be deduced. The internal structure of the nucleon is the main focus of the Hermes experiment at DESY in Hamburg, Germany.

At Hermes, experiments are performed by scattering electrons or positrons from the 27.6 GeV HERA beam on a gaseous target. The simplest target gas to use in order to discover details of the nucleon’s structure is Hydrogen, either in the form of $^1\text{H}$ or $^2\text{H}$. The scattered and produced particles are detected in a forward spectrometer. The electron/positron beam is longitudinally polarized, meaning that the spin of the particle from the beam is aligned or anti-aligned with its momentum. The virtual photon that is exchanged between the beam particle and the target nucleon is therefore also polarized.

In this paper, we are concerned with the exclusive production of a $\rho^0$ meson from a nucleon, the $eN \rightarrow eN\rho^0$ process. The $\rho^0$ meson is a vector particle with spin one, similar to a photon. The $\rho^0$ meson is particularly interesting because the exchanged virtual photon can also fluctuate into a quark-antiquark pair that scatters from the nucleon. The quark-antiquark pair has the same quantum numbers as the $\rho^0$ meson. This, coupled with the valence quark makeup $\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ of the $\rho^0$, means that it is the most likely meson to be produced in such a scenario — the prevalence of the $\rho^0$ meson in the data makes it a useful experimental tool. Many of the models that describe the production of $\rho^0$ mesons are called Vector Meson Dominance (VMD) models. They describe the process in terms of the virtual photon transitioning into the produced meson and then undergoing rescattering with the target nucleon. They are often inspired by Regge phenomenology, which describes the rescattering process in terms of the exchange of hypothetical Reggeons, hadron-like objects whose spin is not quantised. More recently, models based upon Generalized Parton Distributions (GPDs) have been proposed as able to describe the production mechanism.

The $\rho^0$ is an unstable particle and decays into a $\pi^+\pi^-$ pair, so the process chain becomes $eN \rightarrow eN\rho^0 \rightarrow eN\pi^+\pi^-$. This pion pair, along with the scattered electron or positron is detected in the spectrometer. By tracking the directions and energies of these particles, the event in the spectrometer can be reconstructed; the angular decay of the pion pair along with the momentum of the parent $\rho^0$ can be related directly to the polarization of the $\rho^0$ and the four momentum of the virtual photon can be calculated from the measurements of
the initial and scattered beam particle. Knowledge of the initial polarization of the beam particle, and therefore the virtual photon, and the polarization and the angular distribution of the produced $\rho^0$ meson, can be used to place constraints on the details of the scattering mechanism and the underlying details of the structure of the nucleon target in the context of the models discussed above.

The analysis in this paper concerns the helicity amplitudes of the produced $\rho^0$ meson; the helicity of a particle is the projection of the particle’s spin onto the axis of its momentum. In the case of the analysis presented in this paper, we define the momentum direction in the centre of mass frame of the virtual photon and proton system. The dimensionless observables in the scattering process can be described by the ratios of the amplitudes in the scattering process from each of the possible combinations of spin states of the involved particles. These dimensionless observables can be compared to theoretical predictions. The helicity states of the particles can be determined by performing a fit to the angular distribution of the final state mesons to give ratios of the amplitudes. If all the amplitudes ratios are known, the magnitudes can be extracted using measurements of the kinematic dependence of the cross-section. Absolute values for the helicity amplitudes cannot be extracted because some information on nucleon spin-flip amplitudes would be needed that cannot be extracted for unpolarized targets.

The notation used for the helicity amplitudes is $F_{\lambda_V, \lambda_\gamma}$, where $\lambda_V$ is the helicity of the produced $\rho^0$ meson and $\lambda_\gamma$ is the helicity of the virtual photon. The leading contribution to the process is $F_{00}$ where both the virtual photon and produced $\rho^0$ meson are longitudinally polarized. The helicity amplitude ratios are presented in comparison to this leading contribution, so the ratio $f_{11}$ is $F_{11}/F_{00}$.

Typically, Spin Density Matrix Elements (SDMEs) are extracted from the data in exclusive meson analysis. The SDMEs of the $\rho^0$ meson are elements
of the matrix produced from the multiplication of the ratios of helicity amplitudes with the spin density matrix describing the helicity of the incident virtual photon. Since the polarisation of the virtual photon is known from the beam polarisation, the SDMEs can be re-calculated from the helicity amplitude ratios. A previous paper from HERMES has already shown an extraction of the SDMEs for the data set considered in this paper.

The angular distribution of the $\rho^0$ and $\pi$ mesons are described by three dimensional trigonometric terms. The coefficients of the fit terms correspond to SDMEs, combinations of which can be solved for amplitude ratios. Unlike the SDMEs, the extracted amplitude ratios can be decomposed into ‘Natural Parity Exchange’ (NPE) and ‘Unnatural Parity Exchange’ (UPE). The particles involved in NPE must fulfil the constraint $P = (-1)^J$, where $J$ is the total angular momentum and $P$ is the parity of the exchanged particle, which can be any from the list of particles with spin and parity $0^+, 1^+$ etc. such as the $\omega$ meson. The particles that correspond to ‘Unnatural Parity Exchange’ (UPE) must fulfil the constraint $P = (-1)^{(J+1)}$, i.e. come from the list of particles with spin and parity $0^-, 1^-$ etc. such as the $\pi$ meson. As an example, the amplitude corresponding to a transversely polarized photon and a transversely polarized meson is written $F_{11} = T_{11} + U_{11}$ where $T$ is the NPE contribution and $U$ is the UPE contribution. Note that combinations of anti-aligned polarizations of photons and mesons are related to each other as $F_{-1-1} = T_{11} - U_{11}$. Combining these two relations shows that $T_{11}$ and $U_{11}$ are just linear combinations of $F_{11}$ and $F_{-1-1}$.

The NPE exchange case is sometimes known as gluon exchange. The UPE exchange case is sometimes called quark-exchange. In Regge phenomenology at high energies, the UPE is considered to be disfavoured compared to NPE and so the result of the fit for the NPE terms should be much greater than the UPE terms, which could even be consistent with zero. Examination of the kinematic dependences of the ratios with which the helicity amplitudes contribute to the data can be compared with predictions from the models discussed above to test our knowledge of the underlying physics. Helicity amplitudes are complex numbers, so the phase difference between various amplitude ratios can also be useful for comparison to model predictions.

The selection of events from the HERMES data set for the analysis proceeds along the following lines: Events with exactly two pions and a scattered lepton are considered. The event is kept as a suitable $\rho^0$ production event if the pions fulfil requirements that mark their as having come from a $\rho^0$ parent meson. The trajectories of the initial and scattered leptons are examined to check that they are physically compatible with the production of a $\rho^0$ meson and then the kinematics of the entire event are computed to see if they are consistent with the target proton having stayed intact and in its ground state during the process. If all of these requirements are met, the event is kept as an exclusive $\rho^0$ meson production event. The fit is then performed to the distribution of the angles $\Phi$, which is the angle between the lepton scattering plane and the $\rho^0$ meson production plane, $\phi$, which is angle between the $\rho^0$ meson production plane and the $\rho^0$ meson decay plane and $\Theta$, which is the angle in the $\rho^0$ meson rest frame.
between the direction of the 3-momentum of the $\pi^+$ meson and the direction opposite to that of the 3-momenta of the recoiling nucleon (see Fig. 1).

The underlying uncertainties on the measurement come from various sources: there’s a contribution to the total uncertainty from the statistical precision of the measurement, determined by the number of $\rho^0$ production events in the data; there’s a contribution to the uncertainty from the determination of the recoiling nucleon — HERMES cannot measure the recoiling target for this data set and so there is some contribution to the data set from events where the target nucleon breaks up or is in a resonant state which is simulated by Monte Carlo methods; there’s an uncertainty arising from the description of the $\rho^0$ meson in the Monte Carlo models and finally there’s an uncertainty contribution because some of the amplitude ratios that describe unlikely physical events are not considered in the extraction method. The uncertainties in the extraction of amplitude ratios propagate through to the calculation of SDMEs, but the total uncertainties on the calculated unpolarized values are of similar size to those from an extraction of the SDMES directly and are smaller than those from a direct experimental extraction of the polarized SDMEs.

Selected results are given in this summary: Fig. 2 shows the $Q^2$ dependence of the real and imaginary parts of the ratio $T_{11}/T_{00}$. The amplitude $T_{00}$ describes the transition of a longitudinally polarized virtual photon to a longitudinally polarized meson, whereas $T_{11}$ represent the transition of a transversely polarized
Figure 3: Left: The $Q^2$ dependence of phase difference between the $T_{11}$ amplitude and the $T_{00}$ amplitude. Right: The $Q^2$ dependence of phase difference between the $T_{01}$ amplitude and the $T_{00}$ amplitude. The solid curves represent to a functional form motivated by fits to the amplitude ratios to $T_{00}$. The dashed curves represent limits of one standard deviation in the fit parameters. See text for details.

virtual photon to a transversely polarized meson. The curves on the left panel are from fits to the combined proton and deuteron data; the dashed lines are the limits of one standard deviation in the fit parameter. The choice of functional form is motivated by theoretical consideration. The curves on the right panel follow an arbitrary fit to the data because the forms motivated by theory are found not to describe the data. The solid curve is the result of the fit to both the proton and deuteron data sets; the dashed lines are the limits of one standard deviation in the resultant fit parameter.

The left panel of Fig. 3 shows the phase difference between the $T_{11}$ and $T_{00}$ amplitudes, i.e. the phase difference in the NPE amplitudes relating to transversely polarized photons and mesons and those relating to longitudinally polarized photons and mesons. The fit function displayed on the figure is motivated by the fit functions used to describe Fig. 2. The right panel shows the phase difference between the NPE amplitude relating to transversely polarized photons and longitudinally polarized mesons ($T_{01}$) and the amplitude relating to longitudinally polarized photons and longitudinally polarized mesons $T_{00}$. The curves on the figures represent fits to the data following functions motivated by theoretical considerations and individual fits to the real and imaginary parts of the ratio $T_{01}/T_{00}$. The strong dependence on $Q^2$ is not well-understood, nor are the large values.

An overview of the SDMEs calculated by the fit ratio is given in Fig. 4. Shown in the figure is the reduction in the total uncertainty for the calculation of the SDMEs using the amplitude method, compared to a direct extraction of
Figure 4: The 23 SDMEs extracted from proton data using the amplitude ratio method compared to the values obtained using the direct SDME extraction method. The dramatic improvement in the total uncertainty for the polarized SDMEs using the amplitude method is shown.

The results presented in this paper are generally compatible with the SDME analysis performed on the same data set and are also consistent with results on the same process extracted at higher energies by the H1 collaboration. The outcome of the amplitude ratio analysis shows several results unpredicted by theory: the kinematic dependences of the leading NPE amplitudes are not well described by either the VMD or the GPD models; the phase difference between the leading NPE amplitude ratios is large — much larger than predicted by any model; Unnatural Parity Exchange is confirmed as entering into the description of the process on a statistically significant level. The most interesting and important consequence of this analysis is that it allows model-independent comparisons with theoretical predictions of amplitude ratios — this is not fully possible using the SDME extraction technique. The amplitude ratios predicted by theory have a direct correlation with the approximations made in the model calculations and are, therefore, a much more precise tool than the previously published SDMEs for establishing the direct short-comings of the models.